Mathematics 172

Quiz #18 Name: Key

You must show your work to get full credit.

An South American rodent living on an island in the Amazon lives for three years and its life history is partly summarized by the Leslie matrix

$$A = \begin{bmatrix} 0.0 & 1.5 & 9.0 \\ 0.1 & 0.0 & 0.0 \\ 0.0 & 0.8 & 0.0 \end{bmatrix}$$

We wish to find the stable age distribution and the per capita growth rate. Assume that the stable age distribution is of the form

$$\mathbf{n} = \begin{bmatrix} 1 \\ v_2 \\ v_3 \end{bmatrix}.$$

Then if $\lambda = 1 + r$ we have that

$$A\mathbf{n} = \lambda\mathbf{n}.$$

where

$$\lambda\mathbf{n} = \begin{bmatrix} \lambda \\ \lambda v_2 \\ \lambda v_3 \end{bmatrix}.$$

1. Do the matrix multiplication and find $A\mathbf{n}$.

$$\begin{bmatrix} 0 & 1.5 & 9.0 \\ .1 & 0 & 0 \\ 0 & .8 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ v_2 \\ v_3 \end{bmatrix} =$$

$A\mathbf{n} =$ $\begin{bmatrix} 1.5v_2 + 9v_3 \\ .1 \\ .8v_2 \end{bmatrix}$

2. The vector equation $A\mathbf{n} = \lambda\mathbf{n}$ leads to three scalar equations. What are they?

Equation 1 $1.5v_2 + 9v_3 = \lambda$

Equation 2 $.1 = \lambda v_2$

Equation 3 $.8v_2 = \lambda v_3$

3. Solve for v_2 and v_3 in terms of λ

$v_2 =$ $\frac{.1}{\lambda}$

$v_3 =$ $\frac{.8}{\lambda} v_2 = \frac{(.8)(.1)}{\lambda^2} = \frac{.08}{\lambda^2}$

4. Find the equation satisfied by λ.

From Equation 1 of Part 2.

$1.5v_2 + 9v_3 = \lambda$

by 3. $1.5\frac{(.1)}{\lambda} + 9\frac{(.08)}{\lambda^2} = \lambda$

Mutply by λ^2

$.15\lambda + .72 = \lambda^3$

ie $\lambda^3 - .15\lambda - .72 = 0$

Equation is $\lambda^3 - .15\lambda - .72 = 0$

5. Use your calculator to solve this equation and find λ.

$Y1 = X^3 - .15X - .72$

$X\min = 0$

$X\max = 1.5$

Use 2nd calc 2:Zero to find $\lambda = .9520$

$\lambda =$ .9520

6. Find the stable age distribution:

$$\vec{v} = \begin{bmatrix} 1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{.1}{\lambda} \\ \frac{.08}{\lambda^2} \end{bmatrix} = \begin{bmatrix} 1 \\ .10504 \\ .08827 \end{bmatrix}$$

Percent in stage 1 83.8%

Percent in stage 2 8.8%

Percent in stage 3 7.4%

Total $= 1 + .10504 + .08827 = 1.19331$

proportion in stage 1 $= \frac{1}{1.19331} = .8380$

proportion in stage 2 $= \frac{.10504}{1.19331} = .0880$

proportion in stage 3 $= \frac{.08827}{1.19331} = .07397$

Remark In this case the percapita growth rate is $r = \lambda - 1 = -.048$ so the size of the population is decreasing.