Mathematics 172 Homework

1. A tank with a continous supply of nutrients is used to rase duck weed (which will be used to feed tilapia). One kg of duckweek is intorduced into the tank and grows logistically with $r = .5$(kgs/kg)/day and a carrying capacity of $K = 10$kg.

(a) If $N(t)$ is the number of kilograms of duck weed after t days then write the differential equation that N satisfies. *Answer:* $\frac{dN}{dt} = .5N(1 - N/10)$. Or we could use the notation $N'(t) = .5N(1 - N/10)$.

(b) What is the intial rate of change of the duck weed? (That is what is $\frac{dN}{dt}$ when $t = 0$. *Answer:* When $t = 0$ we have $N = 1$ using this in the equation $\frac{dN}{dt} = .5N(1 - N/10)$ to get $N'(0) = .5(1)(1 - 1/10) = .45$kg/day. That is during the first day about .45kg of duck week is added to the tank.

(c) Estimage the amount of duck week in the tank after one day. *Answer:* From part (b) we saw that during the first day about .45kg was added to that tank. As there was already 1kg this gives the estimate of $N(t) \approx 1 + .45 =$ 1.45kg of duck weed.

(d) Estimage $N(87)$. *Answer:* After 87 days we will have reached the carrying capacity and thus $N(78) \approx 10$.

(e) What is the rate of change of the ammout of duck weed when $N = 7$? *Answer:* When $N = 7$ we get $N' = .5(7)(1 - 7/10) = 1.05$.

(f) What value of N maximizes is the rate of change $N'(t) = \frac{dN}{dt}$ the largest? What is the maximum? *Answer:* It is maximumized when $N = K/2 = 10/2 = 5$. The maximum is $N' = .5(5)(1 - 5/10) = 1.25$.

2. Do Problem 1 over using $r = .4$, $K = 20$ and $N(0) = 25$.