Mathematics 172 Homework

Our main result in the our latest class meeting was

Theorem 1. *If r is a constant, then a function $y(t)$ satisfies the differential equation*

$$\frac{dy}{dt} = ry$$

if and only if it is the form

$$y(t) = y_0 e^{rt}.$$

Here are some problems related to this

1. If N satisfies

$$\frac{dN}{dt} = .35N$$

and $N(0) = 45$, then

(a) Give a formula $N(t)$. *Answer:* $N(t) = N_0 e^{rt} = 45e^{.35t}$.

(b) When does $N(t)$ becove 10,000? *Answer:* Solve $45e^{.35t}$ to get $t = \ln(10,000/45)/.35 = 15.439$

2. If

$$\frac{dP}{dt} = rP$$

where r is a constant and $P(10) = 9.6P(0)$, then find r. *Answer:* The form of P is $P(t) = P_0 e^{rt}$. We wish to solve $P(10) = P_0 e^{10r} = 9.6P_0$ for r. Note the the P_0's cancel and we get the solution $r = \ln(9.6)/10 = .2261$

Here is one that is a big more challenging.

3. If we know that a population grows such that

$$\frac{dN}{dt} = rN$$

and that $N(2) = 120$ and $N(5) = 154$, then find r and the intial population size $N_0 = N_0$. *Answer:* We know the $N(t)$ has the form $N(t) = N_0 e^{rt}$. We also know

$$N(2) = N_0 e^{2r} = 120$$
$$N(5) = N_0 e^{5r} = 154$$

You can divide this (to cancel out the N_0's) to get

$$\frac{154}{120} = \frac{N_0 e^{5r}}{N_0 e^{2r}} = e^{5r-2r} = e^{3r}.$$

So that $r = \ln(154/120)/3 = .08315$ Thus we now know

$$N(t) = N_0 e^{.08315t}.$$

This can now be use to find N_0. For example

$$N(2) = N_0 e^{.08315(2)} = 120$$

This

$$N_0 = \frac{120}{e^{.08315(2)}} = 101.615$$

And therefore

$$N(t) = 101.615e^{.08315t}.$$