Mathematics 172 Homework

The material below is also covered in pages 11–14 of the text.

Let a population of organisms grow with out any constrants. Assume they breed once a year and let N_t be the number of them after t years. Let $\Delta_t N$ be

$$\Delta_t N = N_{t+1} - N_t.$$

This is the change in the size of the population from year t to year $t+1$. We then have

$$\Delta_t N = (\text{number of births in year } t) - (\text{number of deaths in year } t).$$

1. Explain why if the size of the population size doubles, that the number of births doubles. And if the population size doubles triples, then the number of births triples. $\square$

From the last problem we see that the number of births is proportional to the number size of the population and therefore here is a constant b (the per capita birth rate) such that

$$(\text{number of births in year } t) = bN_t$$

2. Explain why if the size of the population size doubles, that the number of deaths doubles. And if the population size doubles triples, then the number of deaths triples. $\square$

As before this gives that there is a constant d (the per capita death rate) such that

$$(\text{number of deaths in year } t) = dN_t$$

3. Combine these equations to get

$$\Delta_t N = (b-d)N_t.$$

We set $r = (b-d)$ and call it ***discrete growth factor***. Then we have

$$\Delta_t N = rN_t.$$

4. Use $\Delta_t = N_{t+1} - N_t$ in $\Delta_t N = rN_t$ to get the equation

$$N_{t+1} = (1+r)N_t.$$

$\square$

We set

$$\lambda = (1+r)$$

and call it ***finite rate of increase***. Using $\lambda = 1 + r$ in $N_{t+1} = (1+r)N_t$ gives

$$N_{t+1} = \lambda N_t.$$

We can use this repeatedly to get a formula for N_t.

$$N_1 = \lambda N_0$$
$$N_2 = \lambda N_1 = \lambda\lambda N_0 = N_0\lambda^2$$
$$N_3 = \lambda N_2 = \lambda N_0\lambda^2 = N_0\lambda^3$$
$$N_4 = \lambda N_3 = \lambda N_0\lambda^3 = N_0\lambda^4$$
$$N_5 = \lambda N_4 = \lambda N_0\lambda^4 = N_0\lambda^5$$

so that we have

$$N_t = N_0\lambda^t.$$

If $r = (b - d) > 0$ (that if the per capita birth rate is greater than the per capita death rate) then $\lambda = 1 + r > 1$ and the population grows exponentially. If $r = (b - d) < 0$ (that if the per capita birth rate is less than the per capita death rate) then $\lambda < 1$ and the population decreases exponentially.

5. A lake is has 100 bass released in it. If the per capita birth rate is 7.3 bass per bass and the per capita death rate is 4.7 bass/bass then

(a) Give a formula for number of bass in t years. *Answer:* $N_t = 100(3.6)^t$.

(b) How long until there are one million bass in the lake? *Answer:* 7.190 years.

(c) How long until there are a billion bass in the lake? *Answer:* 12.58 years.

6. In a new dorm, 15 cockroaches are accidentally introduced. After 4 weeks there are 200 of them. Let N_t be the number of roaches t weeks later.

(a) Find both the discrete growth factor, r, and the finite rate of increase, $\lambda = 1 + r$. *Answer:* $\lambda = (200/15)^{1/4} = 1.911$ and $r = \lambda - 1 = .911$.

(b) Give a formula for N_t. *Answer:* $N_t = 15\lambda^t = 15(1.911)^t$.

(c) How long does in take for the population of roaches to double? *Answer:* 1.070 weeks.

(d) How long until there are a million roaches. *Answer:* 17.15 weeks.