

## Mathematics 172 Homework

To simplify the argument I was trying to make in class today, let assume that we have a material that is crushed by a pressure of  $9,000 \text{ lbs/ft}^2$ . If we have a cube of this material with side length  $s$  ft then the weight will be of the form

$$w = cs^3 \text{ lbs}$$

for some constant  $c$ . If a cube with a side of length 2 ft weighs 500 lbs then we can find  $c$  by solving for it in the following

$$500 = c(2)^3$$

which give

$$c = \frac{500}{2^3} = 62.5$$

and thus

$$w = 62.5s^3$$

The base of this cube will be a square with side length  $s$  and therefore its area will be

$$A = s^2.$$

The pressure on the base will be the weight,  $w = 62.5s^3$  lb, spread over an area of  $A = s^2 \text{ ft}^2$ . Thus the pressure will be

$$\text{Pressure on base} = \frac{w}{A} = \frac{62.5s^3}{s^2} = 62.5s$$

So how big can a cube of this material be? If the pressure on the base is larger than the crushing pressure of the material then it will break. In the case here the crushing pressure is  $9,000 \text{ lbs/ft}^2$ . So the largest cube we can have is when

$$62.5s = \text{Pressure on base} = \text{crushing pressure} = 9,000.$$

We can solve for  $s$  to get

$$s = \frac{9,000}{62.5} = 144 \text{ ft.}$$

Thus the largest cube that can be made of this material without it collapsing is 144 ft.

**1.** The crushing pressure of eastern elm is  $707,040 \text{ lb/ft}^2$ . If a cube of eastern elm with side of size 3 ft weighs 945 lbs then how big can a cube made of eastern elm be before it crushes itself under its own weight?

**2.** The crushing pressure of oak is  $1,064,183 \text{ lb/ft}^2$ . If a cube of oak with side of size 1.5 ft weighs 152 lb then how big can a cube made of oak be before it crushes itself under its own weight?