Mathematics 172 Homework

Read pages 35–37 in the text on discrete logistic growth.
The corresponding equation is

$$N_{t+1} = N_t + rN_t \left(1 - \frac{N_t}{K}\right)$$

where r is the per ***capita growth*** rate and K is the ***carrying capacity***. The equilibrium points are

$$N_* = 0, \qquad \text{and} \qquad N_* = K.$$

Thus if $r = .2$ and $K = 100$ we get

$$N_{t+1} = N_t + .2N_t \left(1 - \frac{N_t}{100}\right).$$

This tell us that if we know the population in year t (that is we know N_t) then we can compute the population the next year (that is we can find N_{t+1}. For example if we start with $N_0 = 120$ then

$$N_1 = 102 + .2(120)\left(1 - \frac{210}{100}\right) = 115.2$$

$$N_2 = 115.2 + .2(115.2)\left(1 - \frac{115.2}{100}\right) = 111.7697$$

$$N_3 = 111.7697 + .2(111.7697)\left(1 - \frac{111.7697}{100}\right) = 109.08465$$

$$N_4 = 109.08465 + .2(109.08465)\left(1 - \frac{109.08465}{100}\right) = 107.1027$$

$$N_5 = 107.1027 + .2(107.1027)\left(1 - \frac{107.1027}{100}\right) = 105.5812$$

1. If a population of birds on an island grows with $r = 2.5$ and $K = 100$, if the initial population size if $N_0 = 80$ then find N_1, N_2 and N_3. *Answer:* $N_1 = 120$, $N_2 = 60$, $N_3 = 120$, $N_4 = 60$ and N_t just keeps bouncing back and forth between 60 (when t is even) and 120 (when t is odd).

2. Consider the system

$$N_{t+1} = .6N_t(1 + .5N_t^4)e^{-N_t}.$$

If $N_0 = 4$ find N_1 and N_2. *Answer:* $N_1 = 5.67052$, $N_2 = 6.072944$, $N_3 = 5.718862$, $N_4 = 6.03685$, $N_5 = 5.755141$. For some larger values of t we have $N_{99} = 5.892203$, $N_{100} = 5.892214$, $N_{1=1}5.892204$ and in this case there is a stable equilibrium points with $N_* = 5.8922$. Thus for any reasonable value of N_0 we will have $N_{100} \approx 5.8922$.