Mathematics 172 Homework

1. A population of yeast is growing logistically with an intrinsic growth rate of $r = .6$ grams/day and a carrying capacity of 50 grams. Let $A(t)$ be the number of grams of yeast after t days.

(a) Write the rate equation for A. *Answer:* $\dfrac{dA}{dt} = .6A\left(1 - \dfrac{A}{50}\right).$

(b) A baker starts using the yeast at a constant rate of 4 grams/day. What is the new rate equation satisfied by A? *Answer:* $\dfrac{dA}{dt} = .6A\left(1 - \dfrac{A}{50}\right) - 4$

(c) What are the equilibrium points for the equation of part (b)? *Answer:* $A = 7.921$ and $A = 42.078$.

(d) Which of these is stable? *Answer:* $A = 42.078$.

(e) Redo this problem when the baker is using the rate at 10 grams/day. What happens? *Answer:* The yeast dies out.

(f) What is the greatest rate that the baker can harvest the yeast without killing it off? *Answer:* 7.5 grams/day.

2. Again we have a population of yeast is growing logistically with an intrinsic growth rate of $r = .6$ grams/day and a carrying capacity of 50 grams. Let $A(t)$ be the number of grams of yeast after t days.

(a) Write the rate equation for A. *Answer:* $\dfrac{dA}{dt} = .6A\left(1 - \dfrac{A}{50}\right).$

(b) A baker starts using the yeast at a constant rate of 20% of the total amount per day. What is the new rate equation satisfied by A? *Answer:* $\dfrac{dA}{dt} = .6A\left(1 - \dfrac{A}{50}\right) - .2A$

(c) What are the equilibrium points for the equation of part (b)? *Answer:* $A = 0.0$ and $A = 33.33$.

(d) Which of these is stable? *Answer:* $A = 33.33$.

(e) Redo this problem when the baker is using the rate at 70% grams/day. What happens? *Answer:* The yeast dies out.

(f) What is the greatest percentage that the baker can harvest each day without killing off the yeast? *Answer:* .6 grams/day. (This problem requires more algebra than most.)