Mathematics 172 Homework assigned Monday, January 13.

In class we talked worked with proportional functions. The functions $f(x)$ and $g(x)$ are ***proportional*** if there is a constant c such that

$$f(x) = cg(x).$$

1. If $f(x)$ is proportional to the square of x, and $f(10) = 5$, then give a formula for $f(x)$. (*Answer:*$f(x) = x^2/20 = .05x^2$.)
2. If $f(x)$ is proportional to the reciprocal of x, what happens to the output, $f(x)$, if the input, x, is doubled? (*Answer:*It is divided by 2.) What happens to the output if the input is tripled? (*Answer:*The output is divided by 3.)

Here is are a couple of basic facts that allows one to recognize if a function is proportional to x or x^2.

Theorem 1. *If $f(x)$ is a continuous function defined on $x > 0$ such that*

$$f(2x) = 2f(x) \qquad \text{and} \qquad f(3x) = 3f(x)$$

then $f(x)$ is proportional to x. That is there is a constant c such that

$$f(x) = cx.$$

Theorem 2. *If $f(x)$ is a continuous function defined on $x > 0$ such that*

$$f(2x) = 2^2 f(x) \qquad \text{and} \qquad f(3x) = 3^2 f(x)$$

then $f(x)$ is proportional to x^2. That is there is a constant c such that

$$f(x) = cx^2.$$

3. Let $P(t)$ be a function defined for $t > 0$ such that doubling the input, t, doubles the output, $P(t)$, and tripling the input triples the output. Assume $P(10) = 30$. Find a formula for $P(t)$. (*Answer:*$P(t) = 3t$)
4. Let $Q(r)$ be a function defined for $r > 0$ such that doubling the input, r, multiples the output, $Q(r)$, by $2^2 = 4$ and tripling the input multiples the output by $3^2 = 9$. Assume $Q(2) = 24$. Give a formula for $Q(r)$. (*Answer:*$Q(r) = 6r^2$.)

Consider a triangle and the triangles that have side lengths twice and three times as those in the original triangle. See Figure 1.

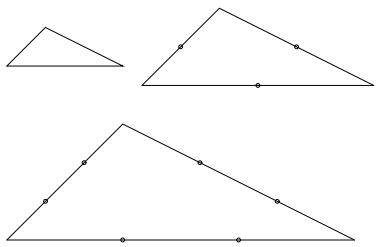

FIGURE 1

5. Recall that the perimeter of a triangle is the sum of the side lengths. How do the perimeters of the triangles in Figure 1 compare?

6. Consider a triangle similar to our original triangle. Assume that the longest side of the original triangle is 2 and that its perimeter is 5. Then what is the perimeter a triangle similar to this triangle where its longest side is 15?

7. What happens to the area of the triangle if we double or triple all the side lengths? See Figure 2 for a hint.

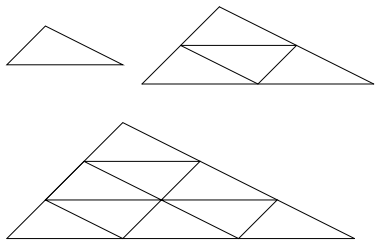

FIGURE 2

8. If the original triangle has area 5 and its shortest side has length 3, then what is the area of a similar triangle whose short side is 30?