Mathematics 172 Test #3 Name: key

You are to use your own calculator, no sharing.
Show your work to get credit.

(1) (10 points) A 12in trout weighs .78 pounds. Predict the weight of a 30in trout.

Weight is proportional to (length)^3 so

Predicted weight. 12.1875 lbs

$W(L) = cL^3$

$W(12) = c(12)^3 = .78$

Thus $c = \frac{.78}{(12)^3}$.

so $W(L) = \frac{.78}{(12)^3} L^3$

Therefore $W(30) = \frac{.78}{(12)^3}(30)^3$

$= 12.1875$

(2) (10 points) Let $f = f(V, P)$ depend on P and Q in such a way that if P is doubled, then f is doubled, if P is tripled, then f is tripled, etc. Likewise assume that if V is doubled, then f is doubled, if V is tripled, then f is tripled, etc. It is known that $f(5, 6) = 3.7$.

(a) Give a formula for $f(P, V)$. $f(P, V) =$ .1233PV

$f(P, V) = cPV$

$f(5, 6) = c(5)(6) = 3.7$

$c = \frac{3.7}{(5)(6)} = .123333$

(b) If $P = 10$ and $f = 25$, then find V. $V =$ 20.27

$f = .1233PV$

$25 = (.1233)(10)V$

$V = \frac{25}{10(.1233)} = 20.27$

(3) (20 points) A white fir is 12 feet tall, has a diameter of .8 feet and weighs 500 lbs. The wood of a white fir crushes at a pressure of 39,000 lbs/ft^2.

(a) Let $D(h)$ be the diameter of a white fir of height h feet. Give a formula for $D(h)$.

Diameter is monotonic to height.

$D(h) = ch$

$D(12) = c(12) = .8$

$c = \frac{.8}{12} =$

$D(h) = .0667h$

(b) Find a formula for the weight, $W(h)$, of a white fir of height h feet.

Weight is monotonic to (height)^3.

$W = ch^3$

$W(12) = c(12)^3 = 500$

$c = \frac{500}{(12)^3} = .2894$

$W(h) = .2894\,h^3$

(c) What is the pressure on the base of a white fir of height h feet?

$P(h) = \frac{\text{weight}}{\text{area}}$

$\text{Area} = \pi(\text{radius})^2$

$= \pi\left(\frac{1}{2}D(h)\right)^2$

$= \pi\left(\frac{1}{2}\right)(.0667h)^2 = .0034906\,h^2$

$P(h) = \frac{W(h)}{\text{Area}} = \frac{.2894\,h^3}{.0034906\,h^2} = 82.89\,h$

Pressure is $82.89\,h$

(d) What is the maximum height of white fir before it crushes itself by its own weight?

Things go bad when

$P(h) = 39{,}000$

i.e. $82.89\,h = 39{,}000$

$h = \frac{39{,}000}{82.89} = 470.5$

Maximum height is 470.5 ft

(4) (30 points) Consider a predator-prey (or a predator-victim) system covered by the equations

$$\frac{dV}{dt} = .1V - .002VP$$
$$\frac{dP}{dt} = -.4P + .0002VP$$

(a) What is the intrinsic grwoth rate of the victims?

Growth rate is .1

(b) What is the death rater of the predators?

Death rate is .4

(c) what is the average number of victims and predators?

Solve $.1 - .002P = 0$

$.002P = .1$

$\hat{P} = \frac{.1}{.002} = 50$

$\hat{V} =$ 2000

$\hat{P} =$ 50

$-.4 + .0002V = 0$

$V = \frac{.4}{.0002} = 2000$

(5) (30 points) Consider the following predator-victim system where in the absence of predators, the prey grows logistically.

$$\frac{dV}{dt} = .015V\left(1 - \frac{V}{300}\right) - .0002VP$$
$$\frac{dP}{dt} = -.2P + .002PV.$$

(a) Find the equilibrium points. $(V, P) =$ (0,0), (300,0), (100,50)

(0,0), (300,0) we set for free.

For the last one solve the system

$.015\left(1 - \frac{V}{300}\right) - .0002P = 0$

$-.2 + .002V = 0$

$\Rightarrow V = \frac{.2}{.002} = 100$

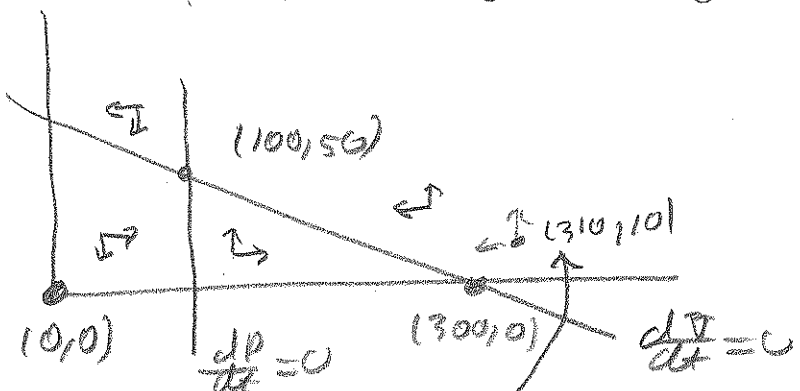
Then

$.015\left(1 - \frac{100}{300}\right) - .0002P = 0$

$.01 - .0002P = 0$

$P = \frac{.01}{.0002} = 50$

(b) Draw the phase diagram showing and labeling the equilibrium points.



(c) If we start at time $t = 0$ with $V = 310$ and $V = 10$

(i) Is the size of the victim population initially increaing or decreasing (circle your answer).

(ii) Is the size of the predator population initially increaing or decreasing (circle your answer).

(d) What happens to the two populations in the long run?

V stabilizes at 100

P stabilizes at 50