Mathematics 172 Test #2 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

(1) (20 points) A species of insect lives for 3 years. The Leslie matrix for its life cycle is

$$L = \begin{bmatrix} 0 & 3.2 & 11.5 \\ .1 & 0 & 0 \\ 0 & .7 & 0 \end{bmatrix}$$

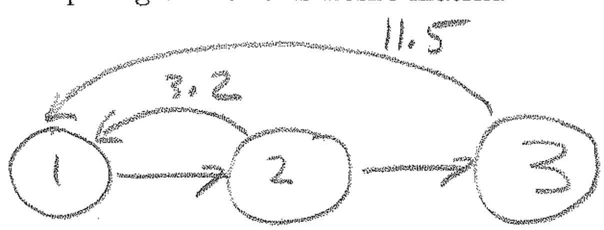
(a) What is the average number of offspring to a three year old?

11.5

(b) What is the proportion of two year olds that survive to become three year olds?

.7 (70%)

(c) Draw the loop diagram for this Leslie matrix.



(d) If we start with

$$\mathbf{n}(0) = \begin{bmatrix} 100 \\ 40 \\ 10 \end{bmatrix}$$

Then what is the initial number of two year olds? 40

(e) Using this value of $\mathbf{n}(0)$ find $\mathbf{n}(50)$.

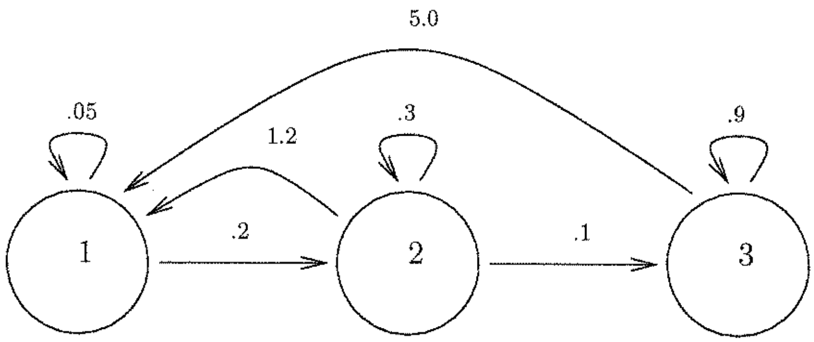
This is done using calculator

$$\mathbf{n}(50) = \begin{bmatrix} 2036.48 \\ 194.94 \\ 130.67 \end{bmatrix}$$

(2) (20 points) A naturalist is taking a census of type of native blackberries. She can distinguish between three stages of the plant.

1. Seedlings,
2. Juveniles,
3. Mature plants.

The life history is summarized by the following loop diagram.



(a) What is the average number of seedlings per year produced by a juveniles

Average number is 1.2

(b) What is the Leslie matrix?

$$L = \begin{bmatrix} .05 & 1.2 & 5.0 \\ .2 & .3 & 0 \\ 0 & .1 & .9 \end{bmatrix}$$

(c) Starting with $\mathbf{n}(0) = \begin{bmatrix} 10 \\ 3 \\ 2 \end{bmatrix}$ compute $\mathbf{n}(40)$ and $\mathbf{n}(41)$ and use these to find the per capita growth rate r. (Be sure to use enough decimal places in your calculations.)

$$\vec{n}(40) = \begin{bmatrix} 262.49 \\ 67.435 \\ 37.798 \end{bmatrix} \quad \vec{n}(41) = \begin{bmatrix} 283.04 \\ 72.722 \\ 40.762 \end{bmatrix}$$

$r =$ .078 (7.8%)

this leads to 3 equations

If $\lambda = 1+r$ we have $\vec{n}(41) = \lambda \vec{n}(40)$

(d) What is the stable age distribution?

For $\vec{n}(40)$ total is

$262.49 + 67.434 + 37.798 = 367.72$

Stable age dist. is

$$\begin{bmatrix} .714 \\ .183 \\ .103 \end{bmatrix}$$

$262.49\lambda = 283.04$

$67.435\lambda = 72.722$

$37.798\lambda = 40.762$

so

$$\lambda = \frac{283.04}{262.49} = \frac{72.722}{67.435} = \frac{40.762}{37.798}$$

$$= 1.078 = 1.078 = 1.078$$

Thus $r = \lambda - 1 = .078$

(3) (20 points)

A species of water bug lives in lakes in Minnesota. We assume that an lake unpopulated by the beetle has a probability of

$$p_i = .3$$

of being colonized by the beetles in a given year and that a populated island has a probability of

$$p_e = .7$$

of having its bug population go extinct in a year.

(a) Using these values of p_i and p_e write the rate rate equation for f, the fraction of the lakes that are populated by the bugs.

$\frac{df}{dt} = p_i(1-f) - p_e f$

in our case $\frac{df}{dt} = .3(1-f) - .7f$

(b) Find the equilibrium points of the rate equation:

Equilibrium points are: .3

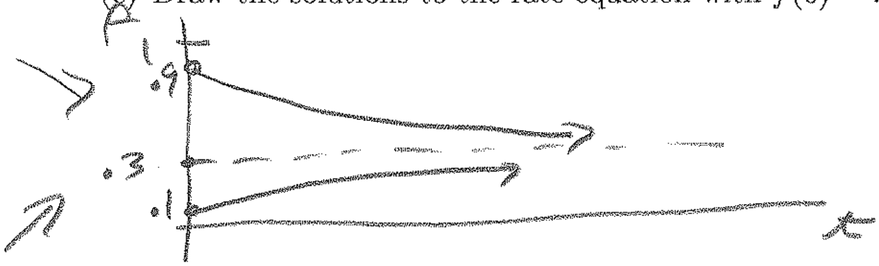
solve

$\frac{df}{dt} = .3(1-f) - .7f = .3 - .3f - .7f = 0$

$-f = -.3$

$f = .3$

(c) Draw the solutions to the rate equation with $f(0) = .1$ and $f(0) = .9$.



(d) Estimate the following:

$f(30) \approx$ .3

$f(50) \approx$ .3

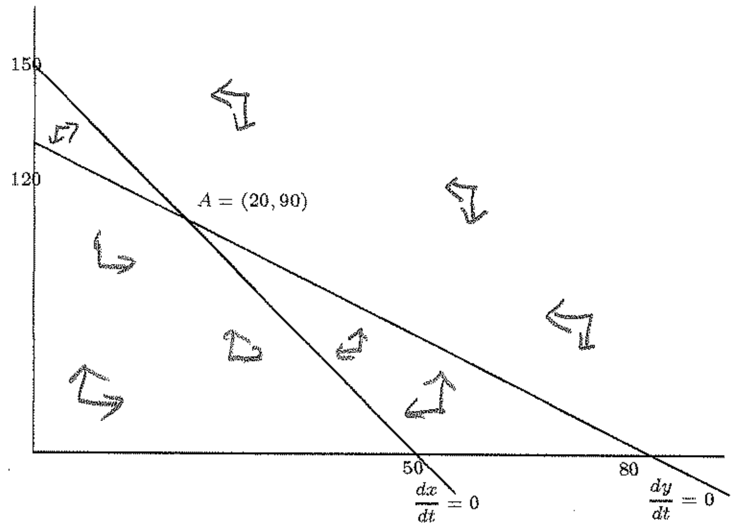
$f(391) \approx$ .3

(e) In the long run what proportion of the lakes do we expect to be populated by the bugs.

.3 (30 %)

(4) (40 points) There are two competing species of fish in pond. Let $x(t)$ be number the first species and $y(t)$ the number of the second species t months after the fish are introduced to the pond.

(a) If the phase diagram for the two species looks like



(i) What is the carrying capacity for the first (that is the x species) when there is none of the second species present.

$K_1 =$ 50

(ii) Is the equilibrium points at A stable? Yes

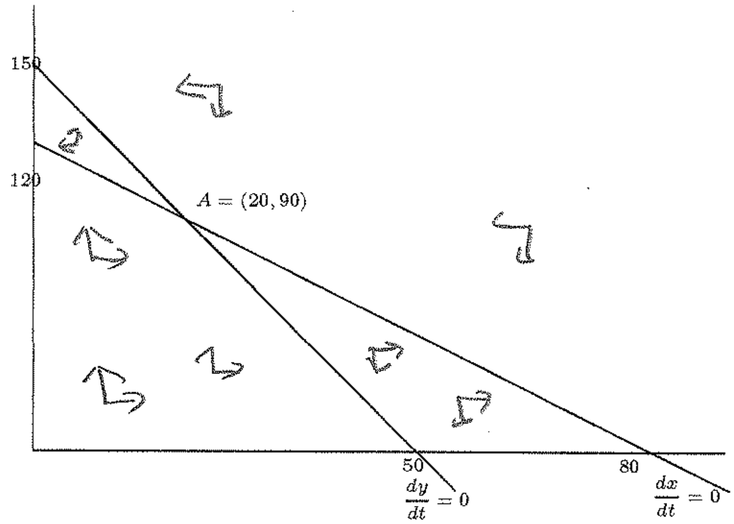
(as all arrows push in towards it.)

(iii) If $x(0) = 5$ and $y(0) = 122$ estimate the following

$x(50) \approx$ 20 $\qquad$ $y(50) \approx$ 90

(iv) In this case is there coexistence or comparative exclusion? (Circle your answer.)

(b) If the phase diagram for the two species looks like



(i) What is the carrying capacity for the first (that is x species) when there is none of the second species present. $K_1 = 80$

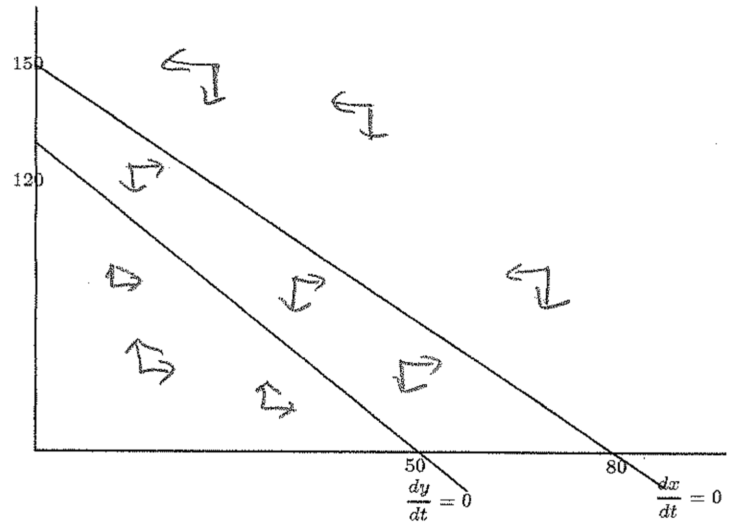
(ii) Is the equilibrium points at A stable? NO

(iii) If $x(0) = 5$ and $y(0) = 122$ estimate the following

$x(50) \approx$ 0 $y(50) \approx$ 150

(iv) In this case is there coexistence or comparative exclusion? (Circle your answer.)

(c) If the phase diagram for the two species looks like



(i) There are three equilibrium points. What are they

(80, 0)

(0, 120)

(0, 0)

(ii) If $x(0) = 30$ and $y(0) = 70$ estimate the following

$x(50) \approx$ 80 $\qquad y(50) \approx$ 0

(iii) What happens to the two populations in this case?

The x population stabilizes at 80
The y population dies out
(ie y population stabilizes at 0)

(5) There are two competing species of crayfish in a small swamp. Let $x(t)$ be the number of species 1 and $y(t)$ the number of species 2 after t months. Assume

$$\frac{dx}{dt} = .1x\left(\frac{200 - x - .25y}{200}\right)$$
$$\frac{dy}{dt} = .07y\left(\frac{600 - 2x - y}{600}\right)$$

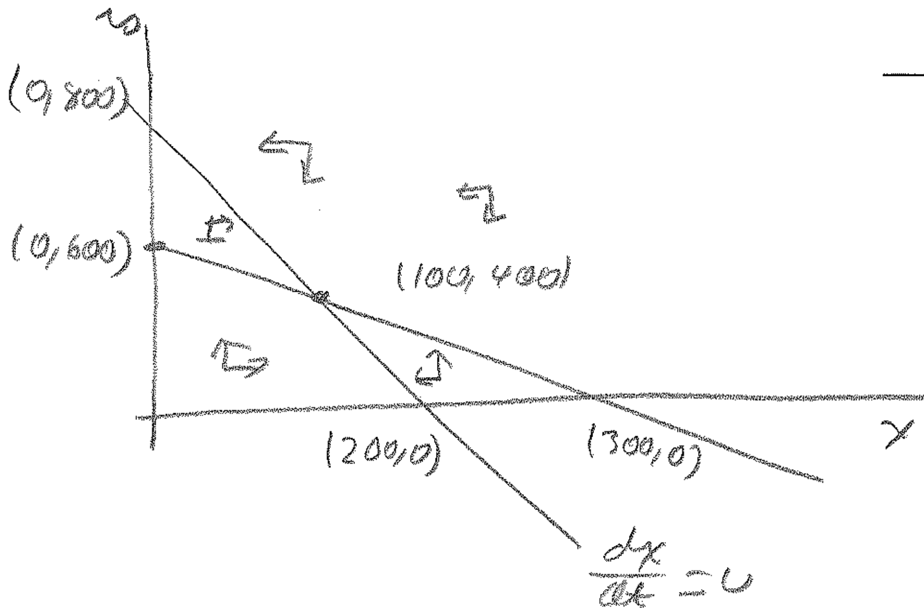
(a) Find all four equilibrium points.

We get 3 for free (0,0), (200,0) (0,600). For the last one solve the system

$x + .2y = 200$
$2x + y = 600$
which gives $x = 100$, $y = 400$

(0,0)
(200,0)
(0,600)
(100, 400)

(b) Which of these equilibrium points are stable?



(100, 400)