Mathematics 172 Test #1 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

(1) (15 points) Ten tigers are released in a national park in India and the population has discrete exponential growth with a per capita rate of $r = .8$ (tigers/year)/tiger.

(a) Write a formula for N_t, the number of tigers after t years.

$N_t = N_0(1+r)^t = 10(1.8)^t$

$N_t =$ $10\,(1.8)^t$

(b) How many tigers are there after five years?

Number of tigers after five years = $188.9 \approx 189$

$N(5) = 10(1.8)^5 =$

(c) How long does it take the population of tigers to reach 1,000?

Time to reach 1,000 7.83 years

We wish to solve

$N(t) = 10(1.8)^t = 1000$

$(1.8)^t = 100$

$t \ln(1.8) = \ln(100)$

$t = \dfrac{\ln(100)}{\ln(1.8)} = 7.83$ year

(2) (15 points) A population of tilapia breeds with continuous exponential growth. If the populations starts with 100 fish and 4 months later has size 150. Find, (a) the intrinsic growth rate r, (b) a formula for the number of fish, $N(t)$ after t months, and (c) how many months it takes for the population size to reach 5,000.

$r =$ $.1014$

$N(t) =$ $100\, e^{.1014t}$

Time to reach 5,000 is 38.59 months

$N(t) = N_0 e^{rt}$

We know

$N(0) = N_0 = 100$

$N(4) = 100e^{4r} = 150$

Thus $e^{4r} = 150/100$

$4r = \ln(150/100)$

$r = \ln(150/100)/4 = .1014$

To find time to 5000 solve

$100e^{.1014t} = 5000$

$e^{.1014t} = 5000/100$

$.1014t = \ln(5000/100)$

$t = \ln(5000/100)/.1014$

$= 38.59$

(3) (15 points) Let $N(t)$ satisfy the rate equation

$$\frac{dN}{dt}(t) = -.3N(N-10)(N-30)$$

(a) If $N(0) = 20$ what if $N'(0)$?

$N'(0) = -.3(20)(20-10)(20-30)$
$= -.3(20)(10)(-10)$
$= 600$

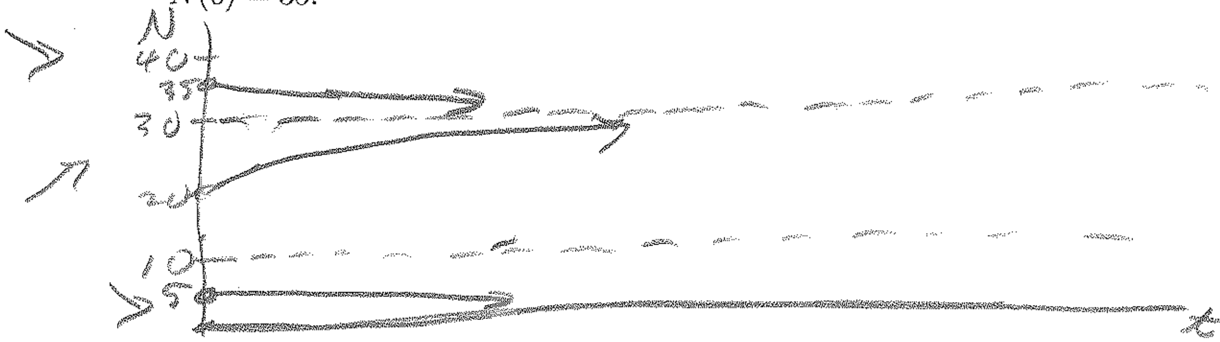
$N'(0) =$ 600

(b) What are the equilibrium points of this equation?

solve
$\frac{dN}{dt} = -.3N(N-10)(N-30) = 0$
to get $N = 0, 10, 30$

0, 10, 30

(c) Sketch the graphs of the solutions the three solutions with $N(0) = 5$, $N(0) = 20$, and $N(0) = 35$.



(d) If $N(0) = 23$ estimate $N(100)$. $N(100) \approx$ 30

starting 23 the graphs goes
to $N = 30$

(4) (15 points) A population of frogs lives on a small island. Let N_t be the number of frogs in year t. Assume that the population grows by the

$$N_{t+1} = f(N_t)$$

where N_t is the number of frogs in year t and the graph of f is given by Figure 1.

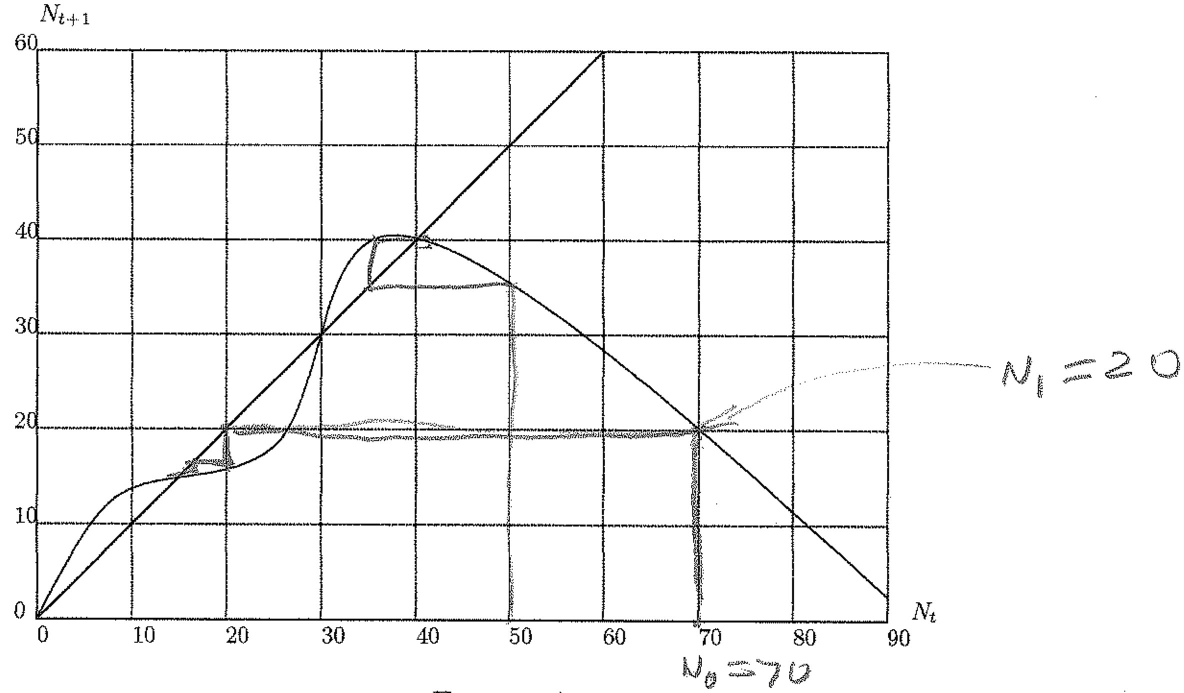


FIGURE 1

(a) If $N_0 = 70$ estimate N_1. $N_1 \approx$ 20

(b) What are the equilibrium points?

This is where the graph crosses the $y=x$ line

Equilibrium points are 0, 15, 30, 40

(c) Which of the equilibrium points are stable? *Explain how you determined they are stable.*

These are were $|\text{slope}| < 1$

Stable equilibrium points 15, 40

(d) If $N_0 = 50$ estimate N_{100} $N_{100} \approx$ 40

From picture we see it spirls into $N=40$

(e) If $N_0 = 70$ estimate N_{100} It goes to $N=15$ $N_{100} \approx$ 15

(5) (15 points) Crayfish are being raised in pond. The population grows logistically with an intrinsic growth rate of $r = .5$ (crayfish/month)/caryfish and a carrying capacity of $K =$ 10,000 caryfish. After the population has become well established the crayfish are harvested at a continuous rate of 500 crayfish/month.

(a) If $N(t)$ is the number of crayfish in the pond t months after the harvesting starts, write a rate equation for $N(t)$.

$$\frac{dN}{dt} = .5N\left(1 - \frac{N}{10{,}000}\right) - 500$$

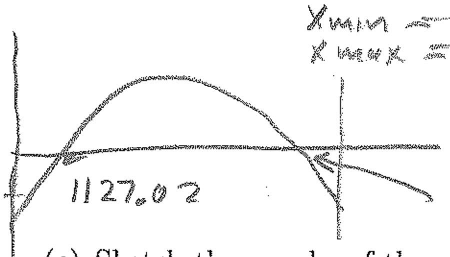
(b) What are the equilibrium points for this equation?

Graph $Y_1 = .5X(1 - X/10000) - 500$

$X_{min} = 0$

$X_{max} = 10000$

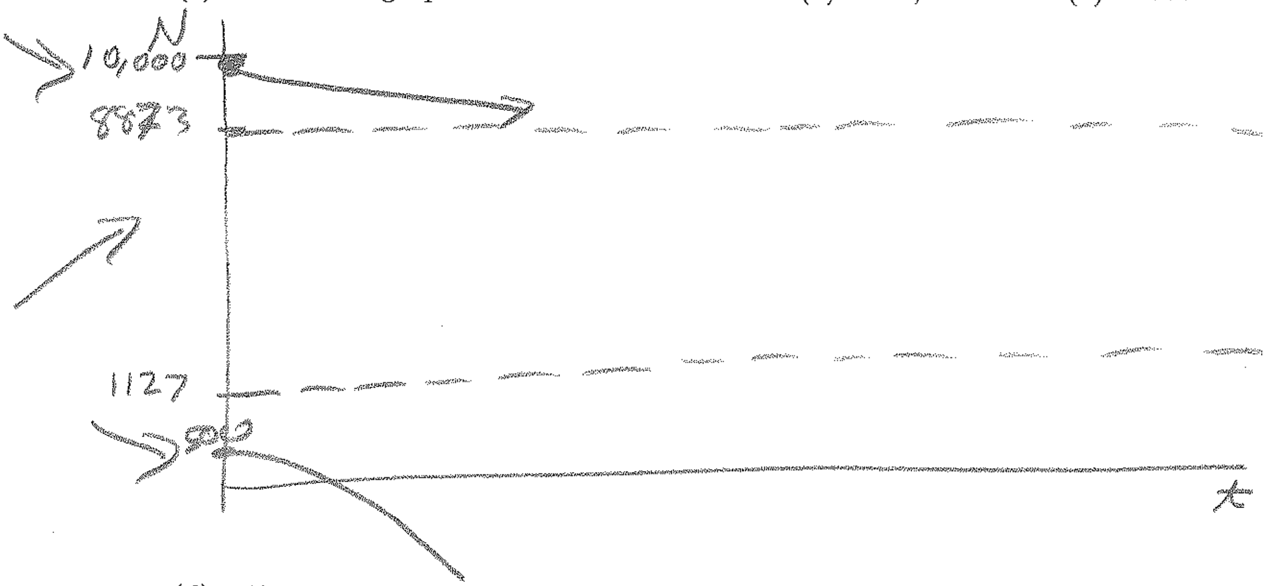
equilibrium points are 1127, 8873



use 2nd calc zero to see the zeros

8872.98

(c) Sketch the graphs of the solutions with $N(0) = 10{,}000$ and $N(0) = 500$.



(d) What is the new stable population size?

Stable population size is 8873

(6) (15 points) Due to fishing pressure, the per capita growth for a population of bass in a lake is reduced by 5% a year. (As bass breed just once a year assume that the growth is discrete exponential.) The South Carolina Department of Natural Resources would like to have a stable population of 25,000 fish in the lake. At what rate should the lake be stocked?

Stocking rate = 1250

Let N_t = population after t years
S = stocking rate.

Then

$$N_{t+1} = N_t - .05N_t + S$$

If $N_t = 25000$, then $N_{t+1} = 25000$ (as this is stable population size)

so

$$\cancel{25000} = \cancel{25000} - .05(25000) + S$$

$$S = (.05)(25000) = 1250$$

(7) (10 points) The size of a population of snails in a backyard pond grows by the rate equation

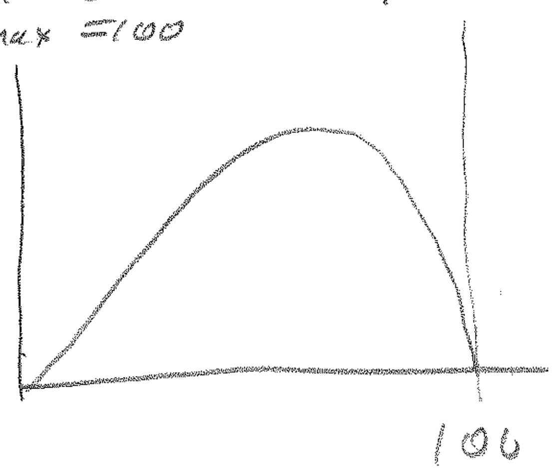
$$\frac{dN}{dt} = .5N\left(1 - \left(\frac{N}{100}\right)^2\right).$$

A some point someone starts to harvest the snails to sell to a pet shop. What is the maximum rate that the snails can be harvested without killing off the population. Explain briefly how you arrived at your answer.

Maximum harvesting rate is 19.245.

We graph $\frac{dN}{dt}$ as a function of N

$$\left(\begin{array}{l} Y_1 = .5X(1-(X/100)^2) \\ X_{min} = 0 \\ X_{max} = 100 \end{array} \right)$$



By 2nd calc max we find $X = 57.735$
$Y = 19.245$

Y is the max. As this is the maximum rate of growth, if we harvest fast than this the population dies off.