Mathematics 172

Name: _____Key_____

You must show your work to get full credit.

The wood of Douglas fir crushes at a pressure of 428.6 lbs/in^2=61,718.4 lbs/ft^2. A Douglas fir of height 10 ft has a diameter at the base of 9 in (= .75 ft) and weighs 350 lbs.

1. Let $D(h)$ be the diameter at the base of a Douglas fir of height h feet. Give a formula for $D(h)$.

$D(h) = ch$

$D(10) = c(10) = .75$

$c = .075$

$D(h) = \underline{\quad .075h \quad}$

2. Let $W(h)$ be the weight of a Douglas fir of height h feet. Give a formula for $W(h)$.

$W(h) = ch^3$

$W(10) = c(10)^3 = 350$

$c = \frac{350}{10^3} = .35$

$W(h) = \underline{\quad .35h^3 \quad}$

3. What is the area of the base a Douglas fir of height h feet?

$A(h) = \pi(radius)^2$

$= \pi(\frac{1}{2}D(h))^2$

$= \pi(\frac{1}{2}(.075h))^2 = .004418h^2$

Area is $\underline{\quad A(h) = .004418h^2 \quad}$

4. What is the pressure on the base of a Douglas fir of height h?

$Pressure = \frac{Weight}{Area}$

$= \frac{.35h^3}{.004418h^2} = 79.22h \ \ lbs/ft^2$

Pressure is $\underline{\quad 79.22 \times h \quad}$

5. What is the maximum height of a Douglas fir before it crushes itself from its own weight.

Things go bad when

$79.22h = 61,718.4 \ lbs$

so maximum weight $= \frac{61,718.4}{79.22} = 779.1 \ ft$

Maximum height is $\underline{\quad 779.1 \quad ft}$

Remark: The largest known Douglas fir was 393 ft tall.