# Homework assigned Friday, January 27.

The following rate equation is sometimes used as a variant on the logistic equation.

$$\frac{dN}{dt} = rN\left(1 - \left(NK\right)^{\theta}\right)$$

where, as usual, $r$ is the intrinsic growth rate and $K$ is carrying carrying capacity. The parameter $\theta$ can be chosen make the equation fit data. Here is a particular case

$$\frac{dN}{dt} = .1N\left(1 - \left(\frac{N}{1,000}\right)^{1.4}\right) \tag{1}$$

**Problem** 1. Draw a graph if $\frac{dN}{dt}$ as a function of $N$ for $0 \leq N \leq 1,200$.

**Problem** 2. What are the stationary solutions? *Answer:* $N = 0$ and $N = 1,000$.

**Problem** 3. If the population is now harvested at a rate of 20 the rate equation becomes

$$\frac{dN}{dt} = .1N\left(1 - \left(\frac{N}{1,000}\right)^{1.4}\right) - 20.$$

Graph $\frac{dN}{dt}$ as a function of $N$ for $0 \leq N \leq 1,000$, use this to find the stationary solutions and graph the solutions with $N(0) = 1000$, $N(0) = 50$ and determine which of the stationary solutions is stable. *Answers:* The stationary solutions are $N = 229.1$ and $N = 818.7$. The stationary solutions $N = 818.7$ is stable.

**Problem** 4. Starting with a population that grows by the rate equation (1) what is the largest harvesting rate that can be used without killing off the population? *Answer:* This largest harvesting rate is the maximum of $.1N\left(1 - \left(\frac{N}{1,000}\right)^{1.4}\right)$ on $0 \leq N \leq 1,000$. Using the graph from problem 1 and the calculator compute this maximum to be 31.23