Homework assigned Friday, January 13.

Let a population of originalisms have a negative intrinsic growth rate r. If $N(t)$ is the size of the population at time t then the rate equation for the growth of the population is

$$\frac{dN}{dt} = rN.$$

Here rN is the natural growth rate of the population. The solution to this is

$$N(t) = N(0)e^{rt}$$

and as r is negative this tends to zero as t gets large. That is the population dies off.

Now assume that the population is stocked at a constant rate, S. Then the rate of change of the population is

$$(\text{rate of change of } N) = (\text{natural growth rate}) + (\text{stocking rate}).$$

Rewritten in mathematical notation this is

$$\frac{dN}{dt} = rN + S$$

For example if the intrinsic growth rate is $r = -.1$ and the stocking rate is

$$S = 500.$$

Then the rate equation is

$$\frac{dN}{dt} = -.1N + 500.$$

To find the stable population size set $\frac{dN}{dt} = -.1N + 500 = 0$, and solve for N to get $N = 500/(.1) = 5{,}000$.

Here are some problems.

Problem 1. A population of algae grows in a pond. If $A(t)$ is the total weight, in kilograms, of the algae at time t (t measured in weeks), then the intrinsic growth rate of the size of the algae population is $r = -.06$. There is water flowing into the pond that adds algae to the pond at a constant rate of 10 kg/wk.

(a) What are the units of r and $\frac{dA}{dt}$?
(b) What is the rate equation satisfied by $A(t)$?
(c) What is the stable population size?

Problem 2. A population of fish in a pond has an intrinsic growth rate of $r = -.2$. DNR (Department of Natural Resources) wants to maintain a stable population size of 5,000 fish. At what rate should they stock the pond to do this?