Homework assigned Wednesday, April 4.

Problem 1. Consider the predator-prey, or in the books terminology, predator-victim system

$$\frac{dV}{dt} = rV\left(1 - \frac{V}{K}\right) - \alpha VP$$
$$\frac{dP}{dt} = -qP + \beta VP$$

where in this case the victim population grows logistically in the absence of the predators. This has three equilibrium points. Solve the equations

$$rV\left(1 - \frac{V}{K}\right) - \alpha VP = 0$$
$$-qP + \beta VP = 0$$

to thing them. *Answer:* They are $(V,P) = (0,0)$, $(V,P) = (K,0)$, and $(V,P) = \left(\frac{q}{\beta}, \frac{r}{\alpha}\left(1 - \frac{q}{\beta K}\right)\right)$.

Problem 2. For the system

$$\frac{dV}{dt} = .2V\left(1 - \frac{V}{200}\right) - .02VP$$
$$\frac{dP}{dt} = -.1P + .001VP$$

Find the equilibrium points, draw the phase plane showing the lines where $\frac{dV}{dt} = 0$ and $\frac{dP}{dt} = 0$ and discuss what happens in the long run. *Answer:* The equilibrium points are $(0,0)$, $(200,0)$ and $(100,5)$. In the long run the populations stabilize at $\widehat{V} = 100$, and $\widehat{P} = 5$. Here is the phase space.

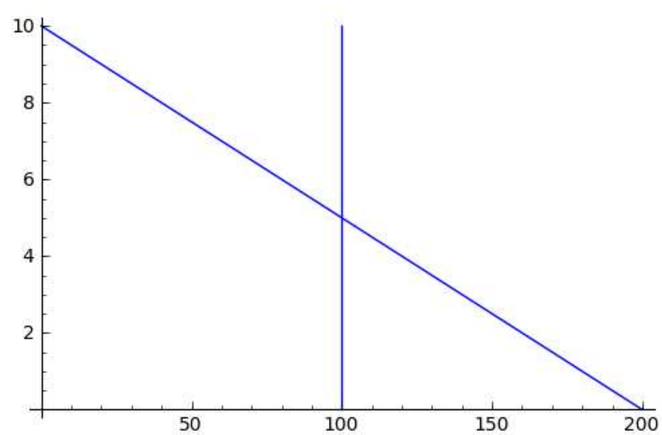

FIGURE 1

Problem 3. Do the same for the system

$$\frac{dV}{dt} = .02V\left(1 - \frac{V}{200}\right) - .002VP$$
$$\frac{dP}{dt} = -.1P + .0004VP$$

The equilibrium points are $(0,0)$, $(200,0)$ and $(250,-2.5)$. But we can ignore the last of these as the negative value $P = -2.5$ has no biological meaning. The phase diagram is

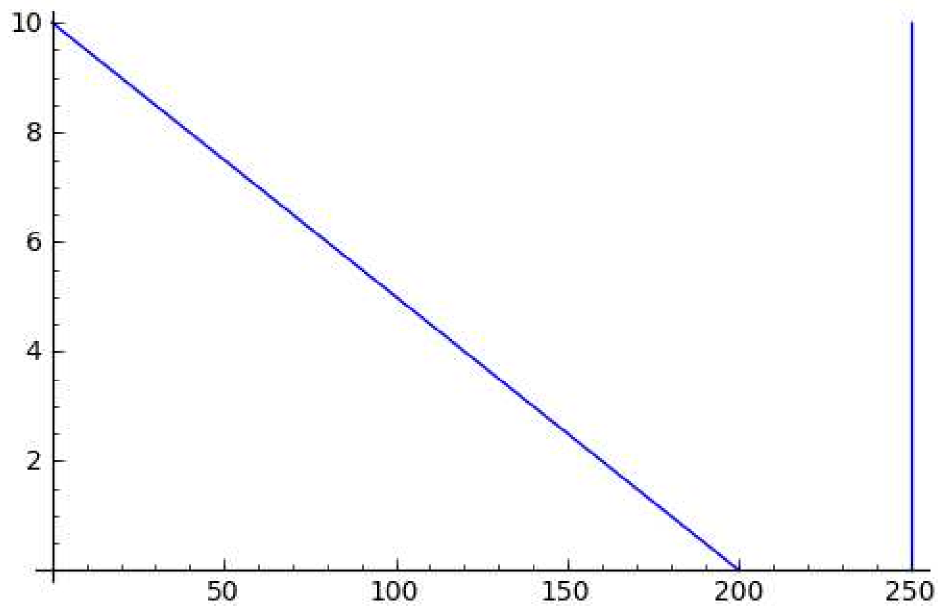

FIGURE 2

In this case the predator goes extinct and the victim population settles down to its carrying capacity $V = 200$.