Homework assigned Monday, March 12.

Our model for competing species is that if

$$x(t) = \text{number of first species at time } t$$
$$y(t) = \text{number of second species at time } t$$

then

$$\frac{dx}{dt} = r_1 x\left(\frac{K_1 - x - \alpha y}{K_1}\right)$$
$$\frac{dy}{dt} = r_2 x\left(\frac{K_2 - \beta x - y}{K_2}\right)$$

We have analyzed the qualitative behavior in terms of the phase diagram. We now want to be a little more qualitative. Let's look at an example.

$$\frac{dx}{dt} = .02x\left(\frac{50 - x - .333y}{50}\right)$$
$$\frac{dy}{dt} = .05y\left(\frac{60 - .5x - y}{60}\right)$$

The equilibrium points are where both $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} = 0$. Note $\frac{dx}{dt} = 0$ implies

$$x = 0 \qquad \text{or} \qquad x + .2y = 40$$

and $\frac{dy}{dt} = 0$ implies

$$y = 0 \qquad \text{or} \qquad .4545x + y = 50.$$

So we find the four equilibrium points by considering four cases.

Case 1. $x = 0$ and $y = 0$. This leads to the equilibrium point $(x, y) = (0, 0)$. Biologically this corresponds to case where there are none of either species. This is an equilibrium points, but it is uninteresting and unstable.

Case 2. $x = 0$ and $.4545x + y = 50$. This gives the equilibrium point $(x, y) = (0, 50)$. Biologically this corresponds to there being none of the first species and the second species growing logistically with carrying capacity $K_2 = 50$.

Case 3. $x + .2y = 40$ and $y = 0$. This gives the equilibrium point $(x, y) = (40, 0)$. Biologically this corresponds to there being none of the second species and the second species growing logistically with carrying capacity $K_1 = 40$.

Case 4. $x + .2y = 40$ and $.4545x + y = 50$. Solving these equations gives $(x, y) = (33, 35)$ (accurate to two decimal places).

We now draw the phase diagram.

By drawing in the arrows for the diagram we see that the equilibrium point at $(33, 35)$ is stable.

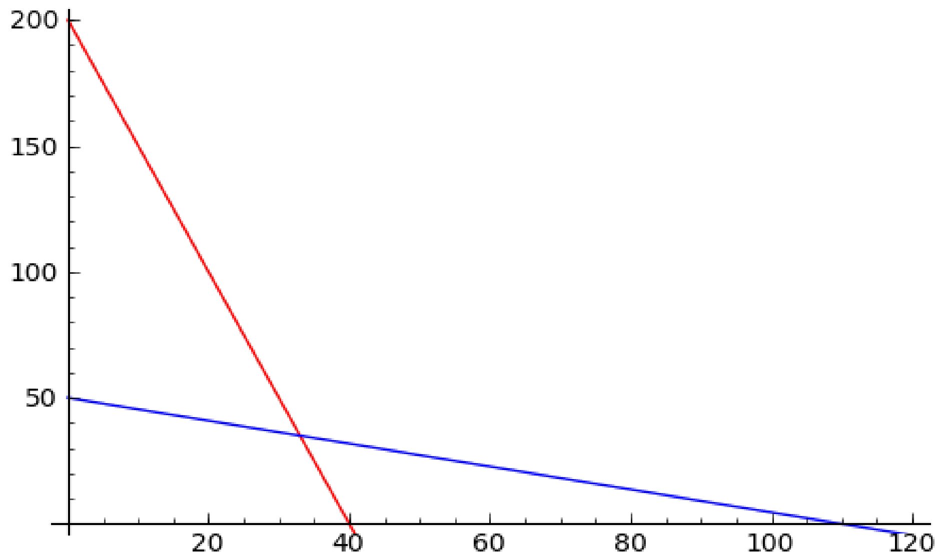

FIGURE 1. The line for $\frac{dx}{dt} = 0$ is in red and the line for $\frac{dy}{dt} = 0$ is in blue.

The point of all this is that we can find the coordinates of the equilibrium points.

Problem 1. Find the coordinates of the equilibrium points of

$$\frac{dx}{dt} = .31x\left(\frac{50 - x - .333y}{50}\right)$$
$$\frac{dy}{dt} = .12y\left(\frac{60 - .5x - y}{60}\right)$$

Answer: $(x, y) = (0, 0)$, $(x, y) = (50, 0)$, $(x, y) = (0, 60)$, and $(x, y) = (36, 42)$.