Mathematics 172 Test #3 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

(1) (50 points) In this problem we have competition between two species of plants governed by the Lotka-Volterra equations

$$\frac{dx}{dt} = r_1 x \left(\frac{K_1 - x - \alpha y}{K_1}\right)$$
$$\frac{dy}{dt} = r_2 y \left(\frac{K_2 - y - \beta x}{K_2}\right)$$

(a) If the graphs for the equations looks like:

Everything ends up here (0,50)

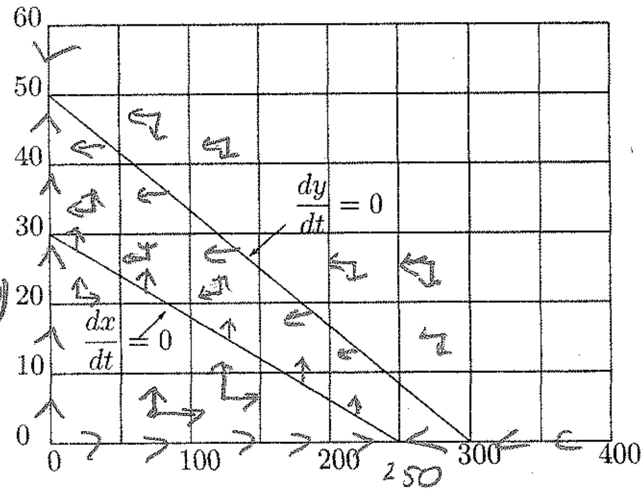


FIGURE 1

(i) What is the carrying capacity for the x-species with the absence of the y-species?

250

(ii) What is the carrying capacity for the y-species with the absence of the x-species?

50

(iii) Draw arrows in the figure showing the directions that x and y are changing.

(iv) What is the long term behavior of the system?

$x(\text{big}) \approx$ 0

$y(\text{big}) \approx$ 50

(v) Is coexistence between these two species possible?

NO

(species x dies off.)

(b) If the graphs for the equations looks like:

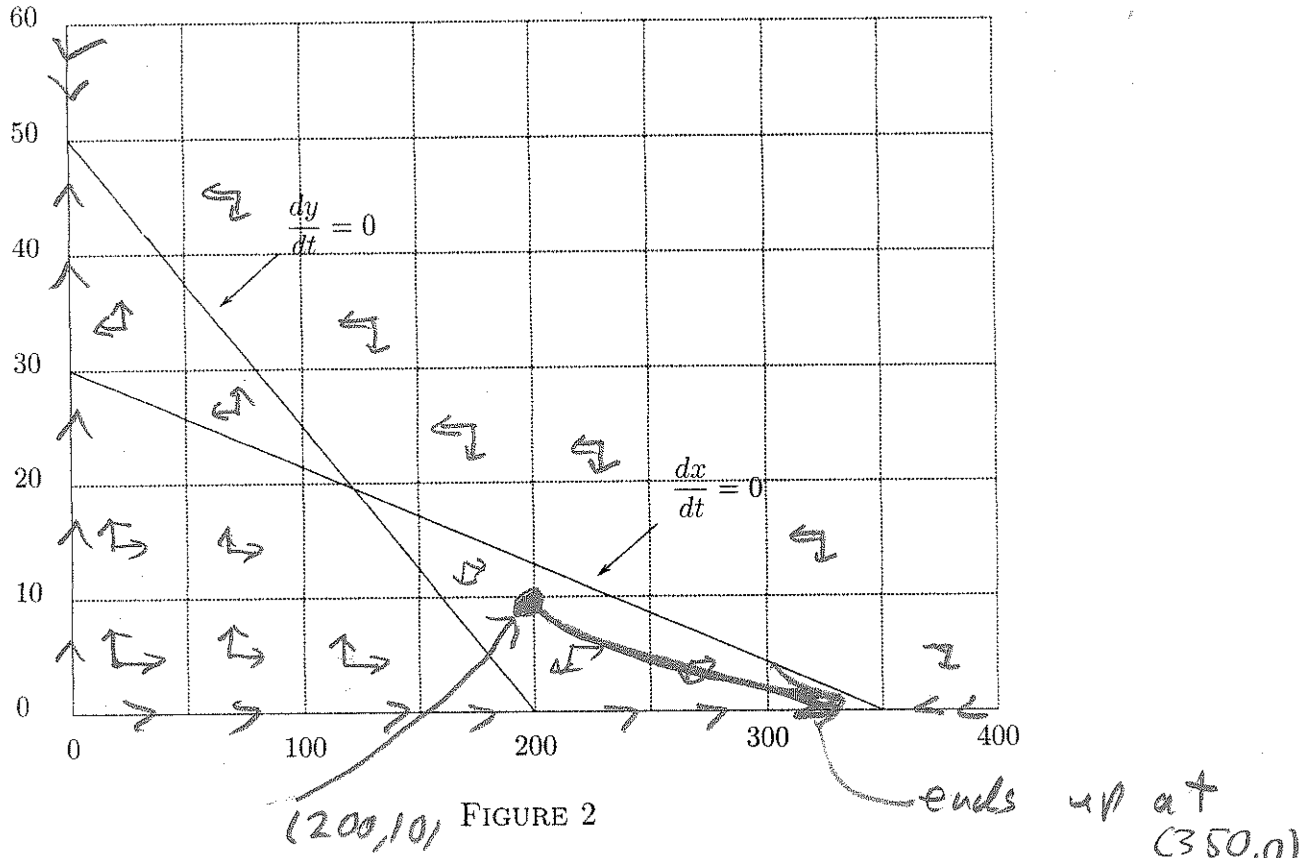


FIGURE 2

(i) Draw arrows in the figure showing the directions that x and y are changing.

(ii) Is coexistence between these two species possible?

NO

(one will out compete the other)

(iii) If $x(0) = 200$ and $y(0) = 10$ what is the long term behavior?

$x(\text{big}) \approx$ 350

$y(\text{big}) \approx$ 0

(2) (25 points) For two competing species of small fish in a pond we have the equations Lotka-Volterra equations

$$\frac{dx}{dt} = .072x\left(\frac{180 - x - .5y}{180}\right) = 0 \qquad x = 180, \quad y = \frac{180}{.5} = 360$$

$$\frac{dy}{dt} = .12y\left(\frac{120 - y - .4x}{120}\right) = 0 \qquad x = \frac{120}{.4} = 300, \quad y = 120$$

(a) Draw a picture including some arrows.

(b) What is the long term behavior of this system? (This will including saying what x(big) and y(big) are? $x(\text{big}) \approx$ 150

$y(\text{big}) \approx$ 60

$x + .5y = 180$

$.4x + y = 120$

$\times 2$: $2x + y = 360$

$-.16x = -240$

$x = \frac{240}{.16} = 150$

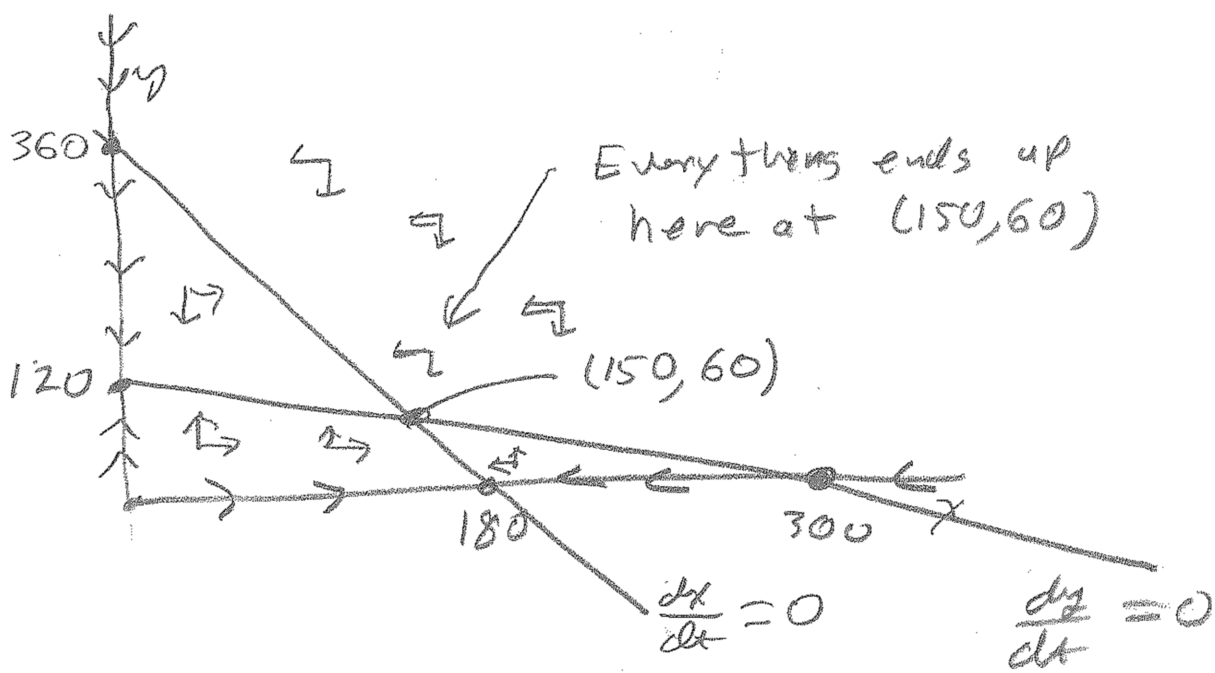
$y = 120 - .4x$

$= 120 - .4(150)$

$= 60$

(c) Is coexistence possible for this system?

Yes



(3) (25 points) For the predator prey equations

$$\frac{dV}{dt} = .15V - .005VP = V(.15 - .005P)$$
$$\frac{dP}{dt} = -.1P + .001VP = P(-.1 + .001V)$$

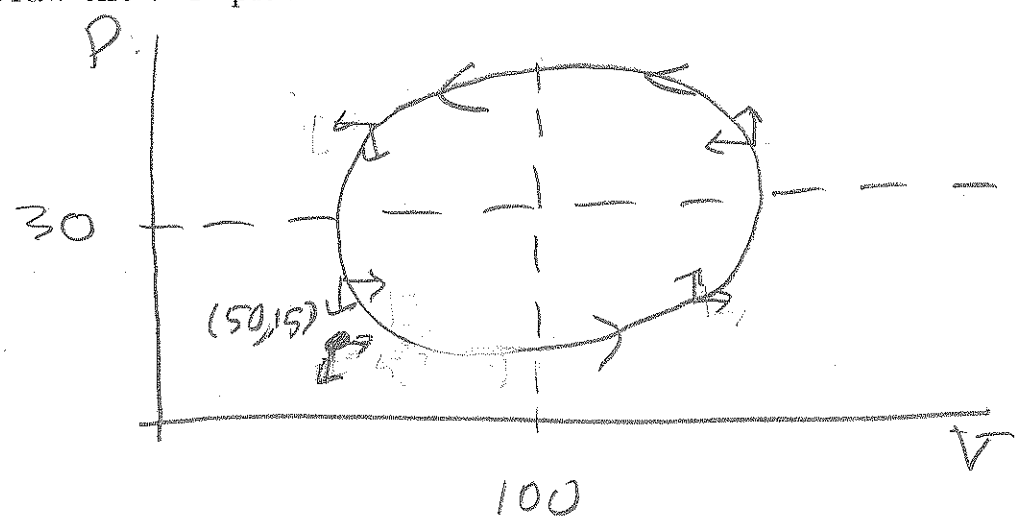
(a) What is the intrinsic growth rate of the victims (with out the presence of prey)?

$r = .15$ ________________

(b) What is the death rate of the predators (when there are no victims for food)?

$g = .1$ ________________

(c) Draw the V-P plane with some arrows included.



(d) In the long run what is the average number of predators and prey?

$.15 - .005P = 0 \quad P = \frac{.15}{.005} = 30$ $\quad V_{\text{Average}} \approx$ 100

$P_{\text{Average}} \approx$ 30

$-.1 + .001V = 0, \; V = \frac{.1}{.001} = 100$

(e) If $V(0) = 50$ and $P(0) = 15$, what happens in the short term? (That is does V increase or decrease, and does P increase or decrease?)

at (50,15) $\quad$ V increases or decreases? ________________

P increases or decreases? ________________

so V increasing and P decreasing