Grades on the First Exam.

Here is the information on the first test. 61 people took the exam. The high score was 105 (9 people got this score). The low scores were 19, 41, 48, 50, and 59. The average was 88.28 with a standard deviation of 16.72. The median was 92. The break down in the grades is in the table.

Grade	Range	Number	Percent
A	90–100	34	55.74%
B	80–89	18	29.51%
C	70–79	3	4.92%
D	60–69	1	1.64%
F	0–59	5	8.20%

Warning.

This exam was the easiest one of the term. Therefore if you did not do well on it you are in trouble. The last day to drop the class without getting a WF is Thursday, October 7. Judging from what I have seen in past classes, ***anyone who got below 70 on this exam is very likely better off dropping the course now.***

Mathematics 172 Test #1 Name: Key

You are to use your own calculator, no sharing.

Show your work to get credit.

(1) (15 points) Ten wolves are released in a national park and the population has discrete exponential growth with a rate of $r = 1.5$ (wolves/year)/wolf.

(a) Write a formula for N_t, the number of wolves after t years.

In general the formula is $N_t = N_0(1+r)^t$.

Here $r = 1.5$, $N_0 = 10$

$N_t =$ $10(2.5)^t$

(b) How many wolves are there after five years?

Number of wolves after five years = 977

$N_5 = 10(2.5)^5 = 976.563$

(c) How long does it take the population of wolves to reach 500?

Time to reach 500 4.269 years

We want to solve

$$N_t = 10(2.5)^t = 500$$

$$(2.5)^t = \frac{500}{10} = 50$$

$$\ln(2.5)^t = \ln(50)$$

$$t\ln(2.5) = \ln(50)$$

$$t = \frac{\ln(50)}{\ln(2.5)} = 4.269$$

(2) (15 points) A population of bacterium has continuous exponential growth. If it starts out with 25 bacterium and 2 hours later there are 225 bacterium then find the intrinsic growth rate r and give a formula for the size, $P(t)$ after t hours.

$r =$ 1.09861

$P(t) =$ $25e^{1.09861t}$

In general the solution is

$$P(t) = P_0e^{rt} = 25e^{rt}$$

$$P(2) = 25e^{2r} = 225$$

$$e^{2r} = \frac{225}{25} = 9$$

$$2r = \ln(9)$$

$$r = \frac{\ln(9)}{2} = 1.09861$$

Note: −5 points if set up with $P_t = P_0(1+r)^t$

(3) (15 points) Algae is growing in a polluted pond with continuous exponential grow rate of $r = -.1$ (alga/week)alga. If a stream replenishes the algae at a continuous rate of 500 alga per week, what is the stable population size of the algae population in the pond?

Stable population size = 5000

Let $N(t)$ = number of alga after t weeks.
Then the rate equation is

$$\frac{dN}{dt} = -.1N + 500$$

If N is the stable size, then $\frac{dN}{dt} = 0$. Thus

$$0 = -.1N + 500$$
$$.1N = 500$$
$$N = \frac{500}{.1} = 5000.$$

Note: −3 points if set up using
$N_{t+1} = (1 - .1)N_t + 500$

(4) (15 points) Due to fishing pressure, the intrinsic rate of growth for a population of bass in a lake is $r = -.02$ (fish/year)/fish. (As bass breed just once a year assume that the growth is discrete exponential.) The South Carolina Department of Natural Resources would like to have a stable population of 10,000 fish in the lake. At what rate should the lake be stocked?

Stocking rate = 200

Let N_t = size of bass population after t years.
This satisfies

$$N_{t+1} = (1 + (-.02))N_t + S = .98N_t + S$$

where S is the stocking rate.

At the stable population size:

$$N_{t+1} = N_t = 10{,}000$$
$$10{,}000 = (.98)10{,}000 + S$$
$$S = (1 - .98)10{,}000 = .02(10{,}000)$$
$$= \underline{200}$$

Note: −3 points if set up with
$\frac{dN}{dt} = -.02N + S$

(5) (15 points) Let $y(t)$ satisfy the rate equation

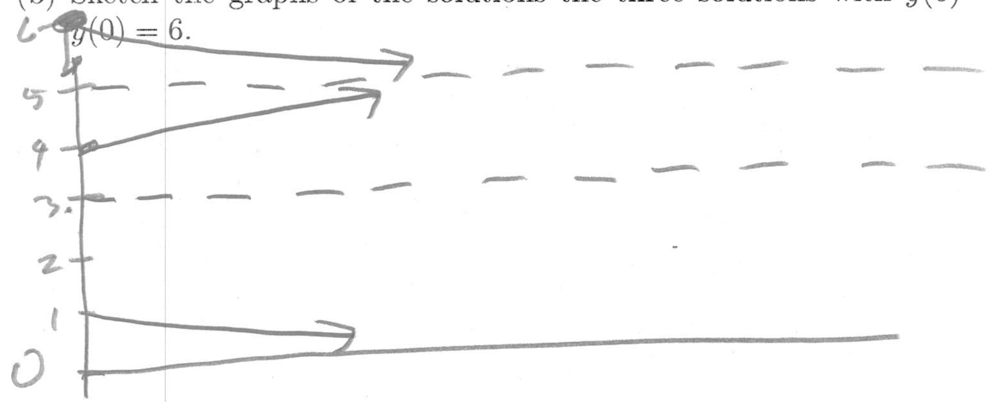
$$y'(t) = -.5y(y-3)(y-5)$$

(a) What are the equilibrium points of this equation?

At equilibrium $y' = 0$
so solve
$-.5y(y-3)(y-5) = 0$

$y = 0, 3, 5$

(b) Sketch the graphs of the solutions the three solutions with $y(0) = 1$, $y(0) = 4$, and $y(0) = 6$.



(c) If $y(0) = 4$ estimate $y(1{,}000)$.

$y(1{,}000) \approx$ 5

(6) (15 points) A population of rabbits grows logistically with a carrying capacity of 500 rabbits and an intrinsic growth rate of $r = 3.5$ (rabbits/year)/rabbit.

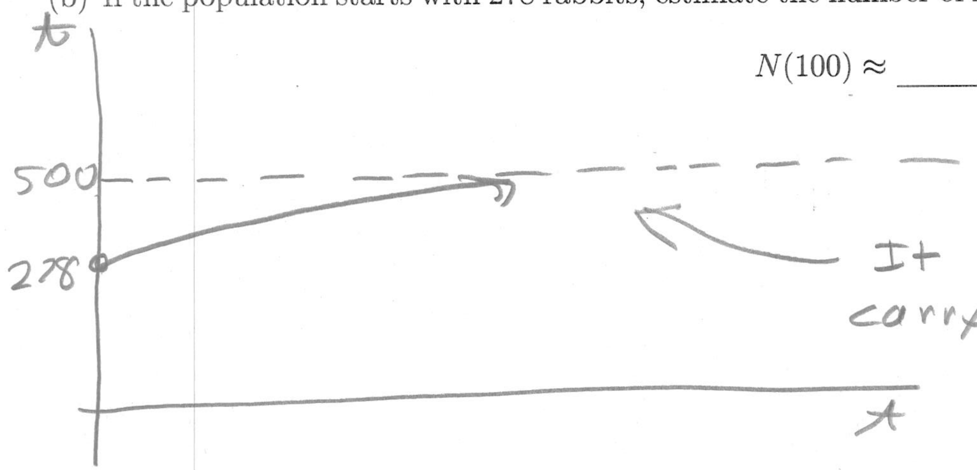
(a) If $N(t)$ is the number of rabbits after t years, what is the rate equation satisfied by $N(t)$? (I.e., just write down the logistic equation.)

Logistic equ is
$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$. In our case $r = 3.5$, $K = 500$

Rate equation is $\frac{dN}{dt} = 3.5N\left(1 - \frac{N}{500}\right)$

(b) If the population starts with 278 rabbits, estimate the number of rabbits after 100 years.

$N(100) \approx$ 500



It goes to the carry capacity = 500

(7) (15 points) An island has a population of birds that grows logistically with an intrinsic growth rate of .2 birds per bird per year and a carrying capacity of 2,000. A bird eating snake is introduced to the island that eats 15% of the current bird population per year.

(a) What it the rate equation for the growth of the bird population after the introduction of the snakes?

Let $N(t) =$ # of birds ~~at~~ years after snakes are introduced.

$$\frac{dN}{dt} = \underbrace{.2N\left(1-\frac{N}{2000}\right)}_{\text{logistic part}} - \underbrace{.15N}_{\text{15\% of current population}}$$

(b) What happens to the stable size of the bird population after the introduction of the snakes?

Stable population size is 500.

Find the equilibrium points by setting $\frac{dN}{dt} = 0$

$$\begin{aligned}
0 &= .2N\left(1-\frac{N}{2000}\right) - .15N \\
&= N\left(.2\left(1-\frac{N}{2000}\right) - .15\right) \\
&= N\left(.2 - \frac{.2N}{2000} - .15\right) \\
&= N\left(.05 - \frac{.2N}{2000}\right)
\end{aligned}$$

solutions are $N = 0$, $N = \frac{(.05)(2000)}{.2} = 500$