Mathematics 172

Quiz #3

You must show your work to get full credit.

A population of fish is over fished until it has a growth rate of $r = -.05$ (fish/year)fish.

(a) If the population is stocked at the rate of 100 fish/year what is the size of the stable population?

(b) At what rate should the population be stocked to give a stable population of 10,000?

Solution for (a): If S is the stocking rate, then population size, N_t, satisfies

$$N_{t+1} = (1 + r)N_t + S.$$

In our case $r = -.05$ and $S = 100$, so this becomes

$$N_{t+1} = .95N_t + 100.$$

When the population reaches its stable size, we have $N_{t+1} = N_t$ and thus

$$N_t = .95N_t + 100$$

which is the same as $.05N_t = 100$ and thus the stable population size is

$$N_t = \frac{100}{.05} = 2{,}000.$$

Solution for (b): The set up is the same, other than we don't know the stocking rate S. So the equation is

$$N_{t+1} = .95N_t + S.$$

To get the stable population size, we set $N_{t+1} = N_t$ and solve:

$$N_t = .95N_t + S$$

which has the solution

$$N_t = \frac{S}{.05}.$$

As we want this to be 10,000 we set

$$\frac{S}{.05} = 10{,}000$$

so the stocking rate is

$$S = (.05)(10{,}000) = 500$$