Work Sheet on discrete dynamical systems

In a discrete dynamical system $N_{t+1} = f(N_t)$ the ***equilibrium points*** (also called a ***stationary point*** or a ***rest point***), N_*, are the points solutions to $f(N) = N$. If $|f'(N_*)| < 1$, then it is a ***stable*** (or ***attracting***) equilibrium points. If $|f'(N_*)| > 1$ it is ***unstable*** (or ***repelling***).

In the special case of the discrete logistic equation with carrying capacity K and per capita grow rate r, that is

$$N_{t+1} = N_t + rN_t\left(1 - \frac{N_t}{K}\right) \tag{1}$$

there will be two equilibrium points, $N = 0$ and $N = K$.

- The equilibrium at $N = 0$ is always unstable.
- The equilibrium at $N = K$ is always unstable is stable when $0 < r < 2$ and unstable when $r > 2$.

Note that the right hand side of this equation also has a zero when $N_t = \dfrac{1+r}{r}K$.

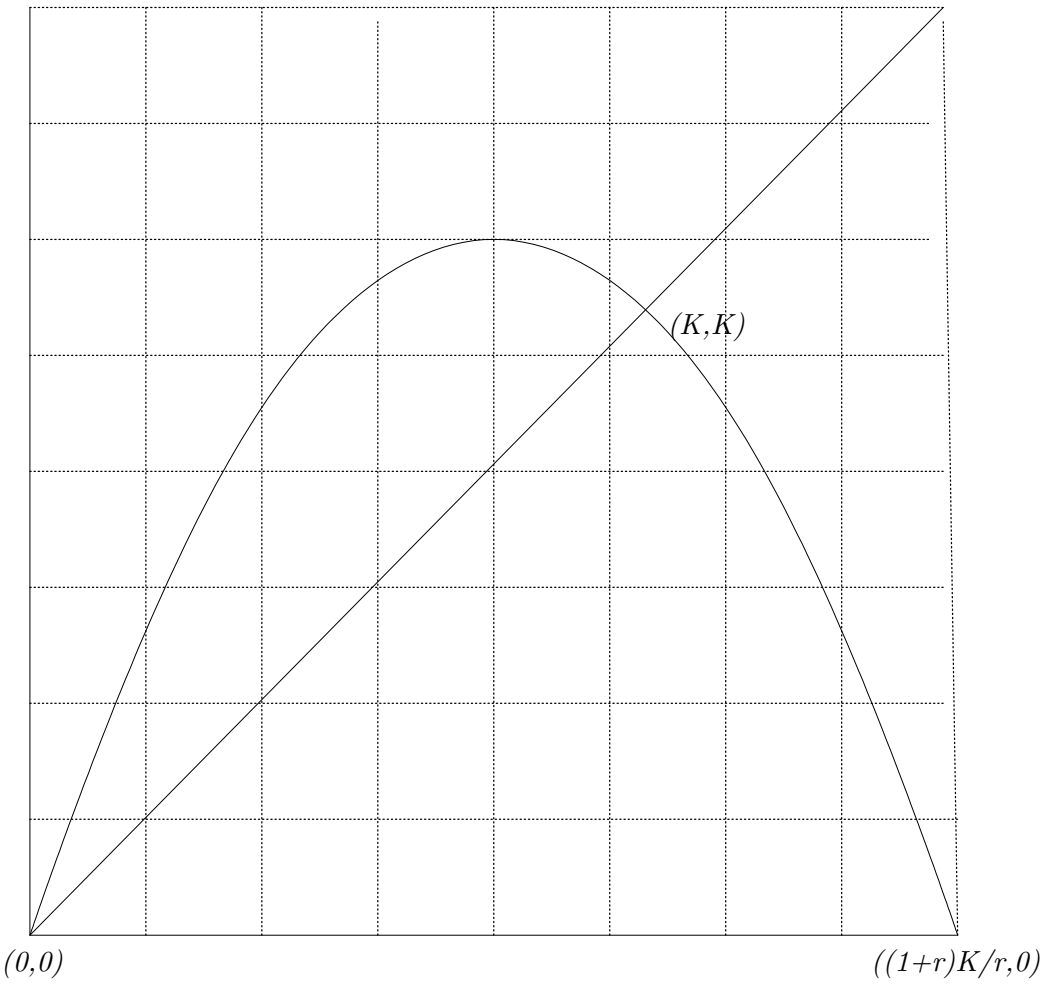

FIGURE 1. The graph of $N_{t+1} = N_t + rN_t\left(1 - \dfrac{N_t}{K}\right)$ as a function of N_t, showing the two equilibrium points $(0, 0)$ and (K, K). Also showing that the zeros of the of the right hand side of (1) are $N = 0$ and $N = \dfrac{1+r}{r}K$.

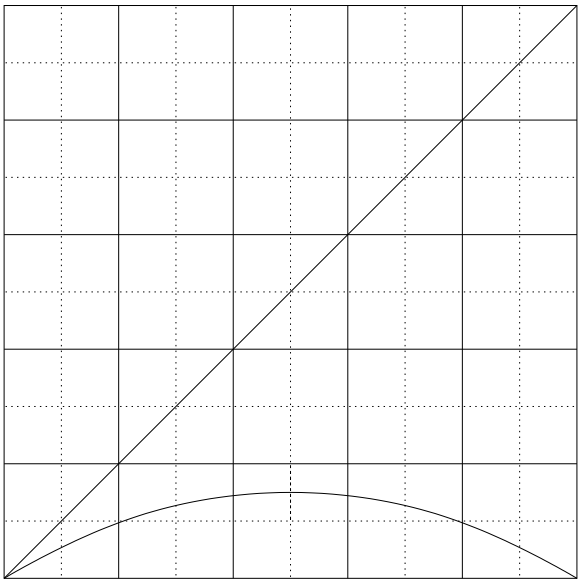

FIGURE 2. In this system there is only one equilibrium point, the one at $N = 0$. Is is stable or unstable? Draw some cobwebs to decide.

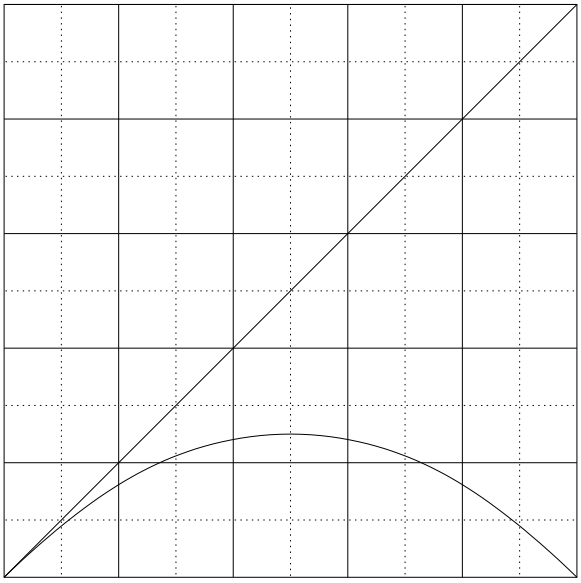

FIGURE 3. Again there is only one equilibrium point, the one at $N = 0$. Is is stable or unstable? Draw some cobwebs to decide.

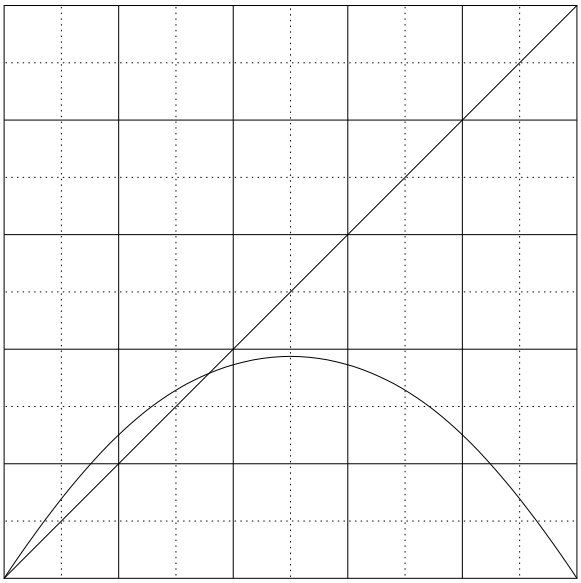

FIGURE 4. Here there are two equilibrium points. Are they stable or unstable?

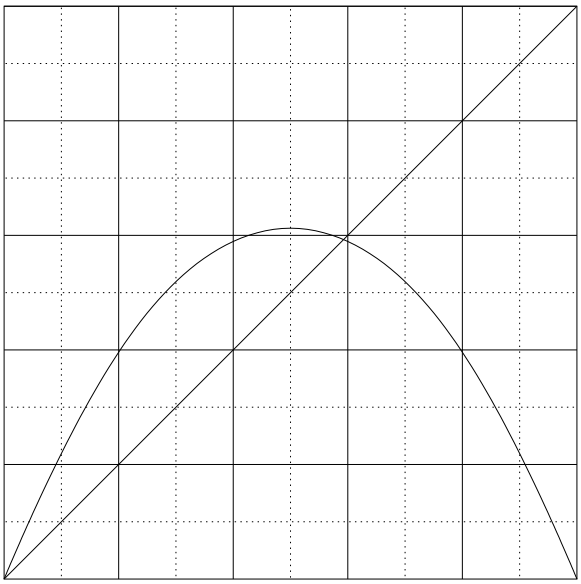

FIGURE 5. Again decide if the two equilibrium points are stable or unstable.

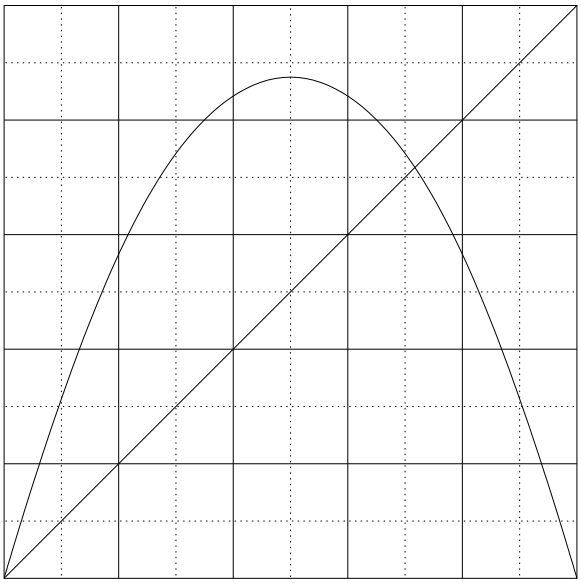

FIGURE 6. Are the two equilibrium points are stable or unstable.

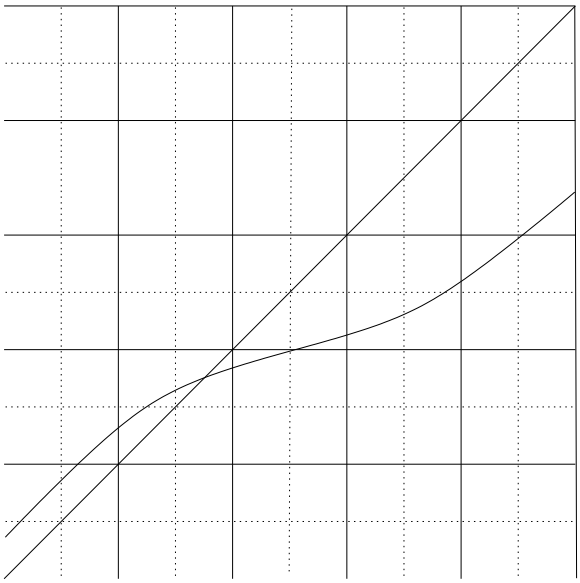

FIGURE 7. Are the two equilibrium point are stable or unstable?

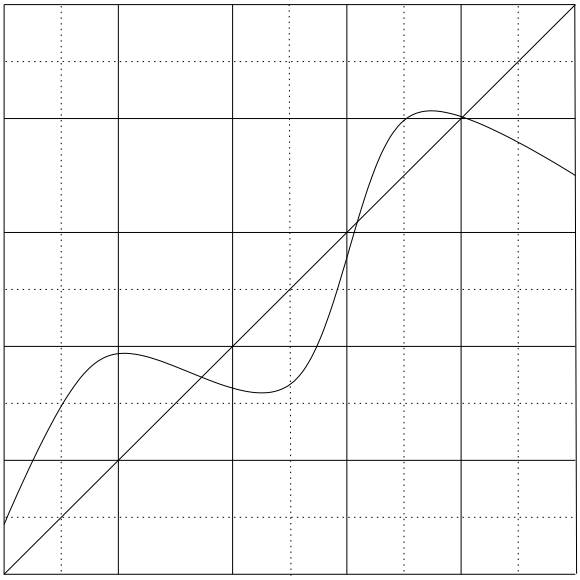

FIGURE 8. Here there are three equilibrium points. Which are stable and which unstable?

Answers:
Figure 2: 0 is stable.
Figure 3: 0 is stable.
Figure 4: 0 is unstable and the other rest point is stable.
Figure 5: 0 is unstable and the other rest point is stable.
Figure 6: Both points are unstable.
Figure 7: The point is stable.
Figure 8: The middle rest point is unstable, the other two are stable.