Work Sheet 9

Recall that for a discrete dynamical

$$N_{t+1} = f(N_t)$$

that the ***equilibrium points*** are the solutions to $f(N) = N$. If N_* is an equibirium point N_* is ***stable*** if $|f'(N_*)| < 1$ and ***unstable*** if $|f'(N_*)| > 1$. (If $|f'(N_*)| = 1$ then no conclusion can be drawn: The point might be either stable or unstable.)

Example for the following

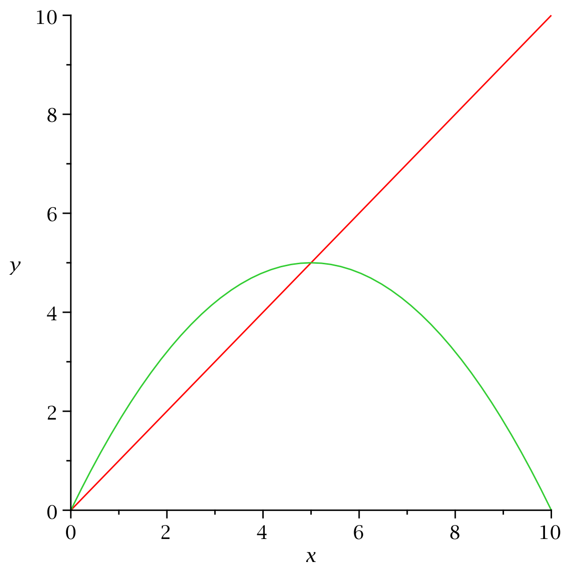

FIGURE 1

find the equilibrium points and use the slope of the graph to decide if they are stable or unstable.

Do the same for the Figure 2.

If none of the equilibrium points are stable, then there will be no stable population size. In this case there are several possibilities.

- It is periodic of order 2. That is there are two values A_1 and A_2 and in the long run the population size bounces back and forth between these two values.
- It is periodic or order 3. In this case there are three values A_1, A_2, A_3 and the population size cycles through these values.
- It is cyclic of order 4, that is there are four values A_1, A_2, A_3, A_4 and the population size cycles through these.
- It is cyclic of order k for some $k \geq 2$ (and you can figure out what this means from the definition just given).

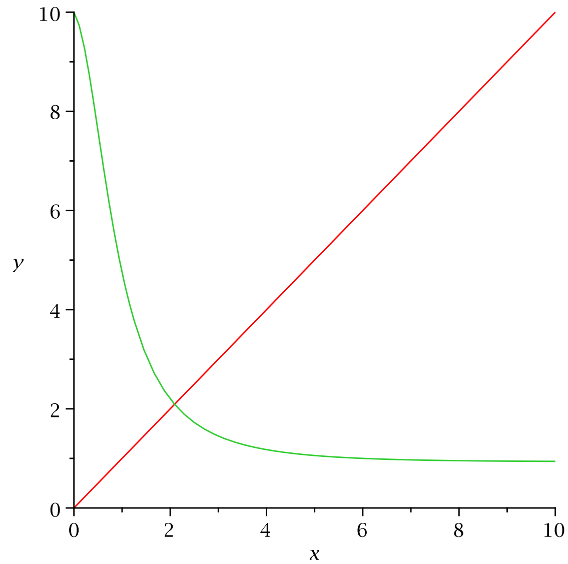

FIGURE 2

- It is not cyclic of any order, but just jumps around some random values. This is called ***chaotic*** behavior.

These options are not exclusive. It may be that for a for a given system $N_{t+1} = f(N_t)$ different values of N_0 can lead to different behavior. For example $N_0 = 10$ might lead to a period of length two, while $N_0 = 15$ leads to chaotic behavior.

For the following find the equilibrium points and classify as to stable or unstable.

(1) $N_{t+1} = f(N_t)$ where the graph if f is given in Figure 3.
(2) $N_{t+1} = 5 + 3^{-.2N_t}$
(3) $N_{t+1} = N_t + .1\left(1 - \dfrac{N_t}{500}\right)$

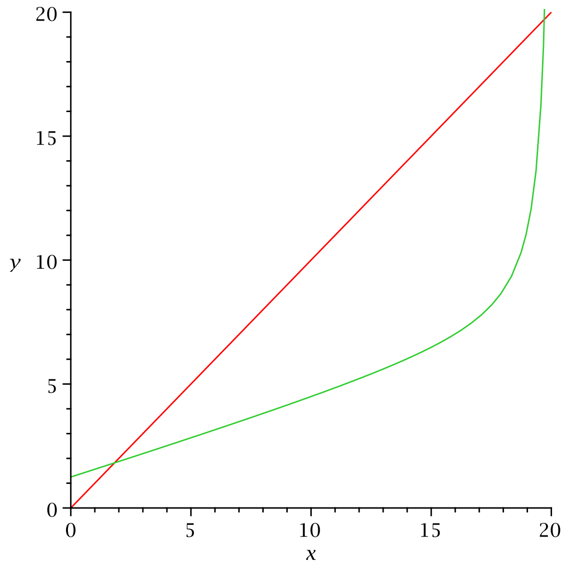

FIGURE 3