Work Sheet 5

The ***logistic equation*** with ***intrinsic growth*** rate ρ (which we assume is positive) and ***carrying capacity*** K is

$$y' = \rho y\left(1 - \frac{y}{K}\right)$$

or, written with respect to the dependent variable P,

$$\frac{dP}{dt} = \rho P\left(1 - \frac{P}{K}\right).$$

One way to think of this is we have an environment that will only support a total of K organisms. We then look for a rate equation of the form

$$\frac{dP}{dt} = r(P)P. \tag{1}$$

where the growth rate $r(P)$ will depend on the size of the population. When the population is small there is almost no constraint on the growth, so $r(0) = \rho$, the intrinsic growth of the organism. When $P < K$ the population will be growing and so we want $r(P) > 0$ for $0 \le P < K$. When $P > K$ the environment is over populated and so the population is decreasing, that is $r(P) < 0$ for $P > K$. There are may functions with this property, but the simplest (in the sense that it is linear) is

$$r(P) = \rho\left(1 - \frac{P}{K}\right)$$

Using this in equation (1) gives

$$\frac{dP}{dt} = \rho\left(1 - \frac{P}{K}\right)P$$

which is the logistic equation.

Note the the two constant functions $P = 0$ and $P = K$ are solutions to the logistic equation. In general we can graph solutions and they look like