Work Sheet 1

If a population currently has a population of P_0 and a per-capita growth rate of r, that is each organism produces (on the average) r new organisms each year (or some unit of time which might be something other than a year) then the size of the population n years in the future is

$$P_n = P_0(1+r)^n.$$

For example, according to Wikipedia, the current population of the USA is $P_0 = 304{,}917{,}000$ and the per-capita growth rate is $r = .97\%$ (which is .0097 offspring per person per year). Thus, if the birth rate, and death rates stay the same (and ignoring factors such as immigration) the population of the USA n years from now will be

$$P_n = P_0(1+r)^n = 304{,}917{,}000(1.0097)^n.$$

Problem 1. According to this formula, what what will be the population of the USA 5 years from now? In 10 years from now? In 20 years from now? In 100 years from now?

Problem 2. Give reasons why these estimates might or might not be reasonable.

The current population of Liberia is 3,489,072 (source: Wikipedia again) and it has the highest per per-capita growth rate of any country at 4.50%.

Problem 3. Give a formula for the population of Liberia n years from now.

Problem 4. According to this formula, what what will be the population of the Liberia 5 years from now? In 10 years from now? In 20 years from now? In 100 years from now?

Problem 5. If these formulas hold, when will the population of Liberia overtake the population of the USA?

Problem 6. Do you believe the following: If there are two populations, the first with initial population P_0 and per-capatia growth rate r, the second with initial population Q_0 and per-capatia growth rate ρ with $r < \rho$ then, regardless of the initial populations (regardless of the values of P_0 and Q_0) that eventual the second population will become the largest. (Restatement: Do you believe that given two populations the one with the largest per-capatia growth rate will eventually become the largest.) Explain your answer.

Homework.

(1) The following table gives values for P and t.

t	0	1	2
P	20	30	45

(a) Explain why this data could be exponential.
(b) What is the value of P when $t = 5$?

(2) A popolation of fish in a pond has a per-capita growth rate of 20% per year.
   (a) How long does take for the population to double?
   (b) If we start with 50 fish, then how many are there after 30 years? After 100 years?

(3) There are populations of two species of rats on an island. There are currectly 100 rats of species A and this species has a per-capatia growth rate of 60% per year. There are currectly 1,000 rates of species B and this species has a per-captia growth rate of 20% per year. How long before the number of rats of species A is larger than the number of rats of species B?

(4) Under the right conditions some types of bacteria will divide every 20 minutes. That is the population will double every 20 minutes.
   (a) Starting with just one bacterium how may are there after one hour? After 12 hours? After a day? After a week? After 30 days? After 365 days?
   (b) The weight of a single bacterium is about 10^{-9}mg. What is the weight of the population of bacteria after 365 days? Compare this with the weight of the Earth which is 5.9736×10^{24}kg.