Work Sheet 8

Grades on the First Exam.

Here is the information on the first test. 56 people took the exam. The high score were 105 (9 people got this score). The low scores were 67, 65, 52, 36, 32, and 27 The average was 87.41 with a standard deviation of 18.67. The median was 92. The break down in the grades is in the table.

Grade	Range	Number	Percent
A	90–100	36	64.29%
B	80–89	6	10.71%
C	70–79	7	12.50%
D	60–69	4	7.14%
F	0–59	4	7.14%

Warning.

This exam was the easiest one of the term. Therefore if you did not do well on it you are in trouble. The last day to drop the class without getting a WF is Thursday, October 2. Judging from what I have seen in past classes, ***anyone who got below 70 on this exam is very likely better off dropping the course now.***

Mathematics 172 Test #1 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

(1) (10 points) Let y satisfy

$$y'(t) = .7y(t), \qquad y(0) = 10$$

(a) What is the solution to this equation? $y(t) = 10e^{.7t}$

$$y(t) = y(0)e^{rt} = 10e^{.7t}$$

(b) When (i.e. for what value of t) does $y(t)$ double? $t = .99021$

we want t so that

$$y(t) = 2\,y(0)$$

i.e. $10e^{.7t} = 2 \cdot 10$

$$e^{.7t} = 2$$

$$.7t = \ln(2)$$

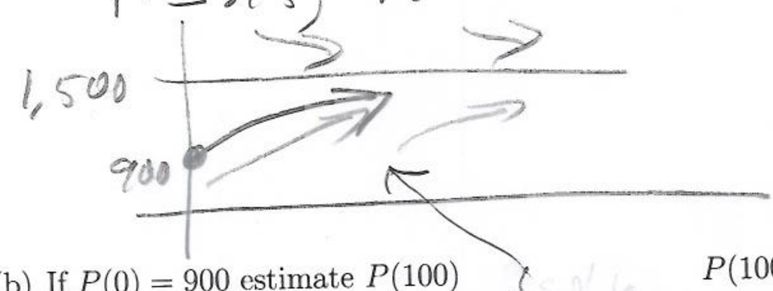
$$t = \frac{\ln(2)}{.7} = .99021$$

(2) (10 points) Let $P(t)$ the size of a population of flies that grows logistically with an intrinsic growth rate of .15 flies per fly per week and a carrying capacity of 1,500.

(a) Write the rate equation for $P(t)$ $\dfrac{dP}{dt} = .15\left(1 - \dfrac{P}{1{,}500}\right)$

$\dfrac{dP}{dt} = rP\left(1 - \dfrac{P}{K}\right)$ where

$r = .15$, $K = 1{,}500$



(b) If $P(0) = 900$ estimate $P(100)$ $P(100) \approx$ 1,500

solution goes to $P = 1500$

(3) (15 points) A population of bacterium grows with a constant intrinsic growth rate. If it starts out with 10 bacterium and 2 hours latter there are 90 bacterium then find the intrinsic growth rate r and give a formula for the size, $P(t)$ after t hours.

$r =$ 1.0986

$P(t) =$ $10 e^{1.0986 t}$

Thus will grow like

$$P(t) = P_0 e^{rt} = 10 e^{rt}.$$

To find r note

$$P(2) = 10 e^{r(2)} = 90$$

$$e^{2r} = 9$$

$$2r = \ln(9)$$

$$r = \frac{\ln(9)}{2} = 1.0986$$

so $P(t) = 10 e^{1.0986 t}$

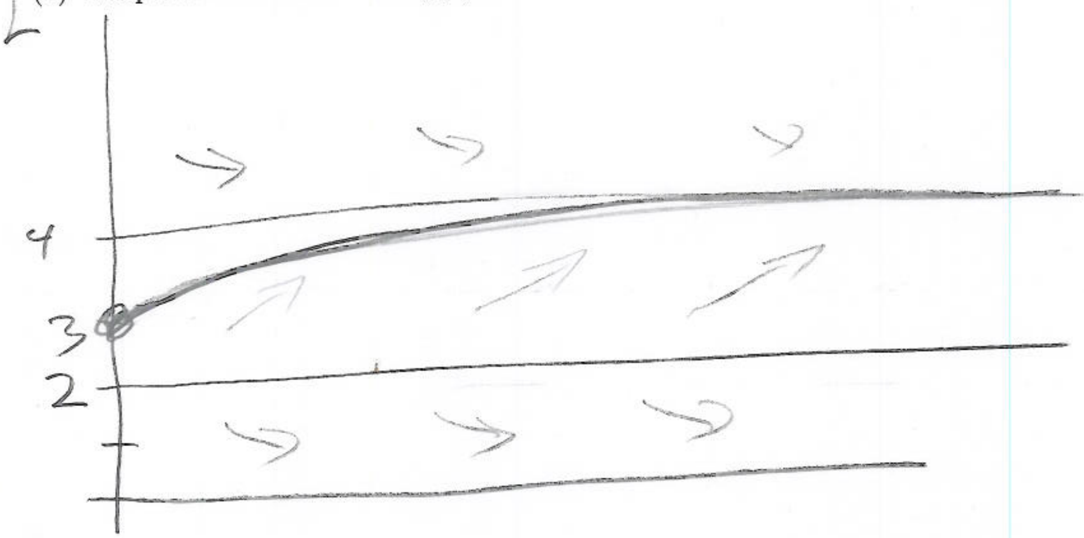
(4) (20 points) Let $y(t)$ satisfy the rate equation

$$y'(t) = -3y(y-2)(y-4)$$

(a) What are the stable equilibrium points? 0, 4

(b) What are the unstable equilibrium points? 2

(c) Graph the solution with $y(0) = 3$



(d) If $y(0) = 3$ estimate $y(100)$. It approaches the asymptote $y = 4$

$y(100) \approx 4$

To find equilibrium points set to set

$-3y(y-2)(y-4) = 0$

$y = 0, 2, 4$

so 0, 4 stable

2 unstable

(5) (15 points) For the rate equation

$$\frac{dP}{dt} = .1(P - 100)$$

do the substitution $y = P - 100$.

(a) What is the rate equation for y?

$$\frac{dy}{dt} = .1y$$

$$\frac{dy}{dt} = \frac{dP}{dt} - 0 = \frac{dP}{dt} = .1(\underbrace{P-100}_{y}) = .1y$$

(b) Find the solution with $P(0) = 80$.

$$P(t) = 100 - 20e^{.1t}$$

The solution to

$$\frac{dy}{dt} = .1y \quad \text{is}$$

$$(*) \quad y(t) = y(0)e^{.1t}$$

But $y(t) = P(t) - 100$

and $y(0) = P(0) - 100 = 80 - 100 = -20$

Use these in $(*)$ to get

$$P(t) - 100 = -20e^{.1t}$$

$$P(t) = 100 - 20e^{.1t}$$

(6) (15 points) Due to fishing pressure, the intrinsic rate of growth for a population of bass in a lake is $r = -.02$. The South Carolina Department of Natural Resources would like to have a stable population of 8,000 fish in the lake. At what rate should the lake be stocked?

Stocking rate = 160 bass/year

Let $P(t)$ = number of bass at time t.

S = stocking rate.

Then the rate equation for $P(t)$

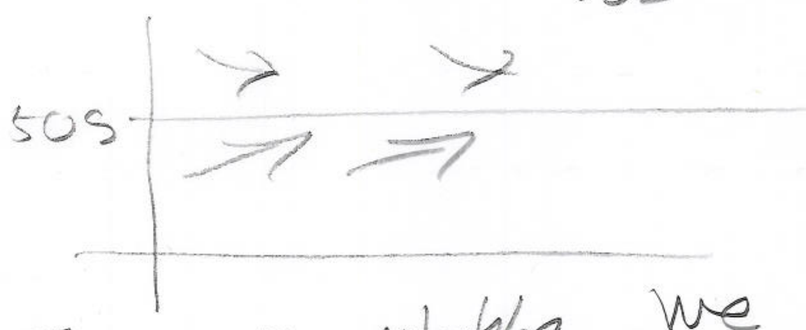
is $\frac{dP}{dt} = -.02P + S$

The equilibrium points is when

$-.02P + S = 0$

so $.02P = S$

$P = \frac{S}{.02} = 50S.$



This is stable. We want

$50S = 8{,}000$

so $S = \frac{8{,}000}{50} = 160$

(7) (20 points) An island has a population of birds that grows logistically with an intrinsic growth rate of .2 birds per bird per year and a carrying capacity of 1,200. A bird eating snake is introduced to the island that eats 15% of the bird population per year.

(a) What it the rate equation for the growth of the bird population after the introduction of the snakes?

$$\frac{dP}{dt} = .2P\left(1 - \frac{P}{1{,}200}\right) - .15P$$

(b) What happens to the size of the bird population after the introduction of the snakes?

To find equlibrium points set

$$.2P\left(1 - \frac{P}{1200}\right) - .15P = 0$$

$$P\left(.2\left(1 - \frac{P}{1200}\right) - .15\right) = 0$$

so $P = 0$ or

$$.2\left(1 - \frac{P}{1200}\right) - .15 = 0$$

$$.2 - \frac{.2P}{1200} - .15 = 0$$

$$\frac{.2P}{1200} = .2 - .15 = .05$$

$$P = \frac{(.05)(1200)}{.2} = 300$$

So the population declines to a stable size of 300 birds.