

Quiz #19

Name: Key

*You must show your work to get full credit.*

For the system

$$\begin{aligned}\frac{dx}{dt} &= x(3-x-y) = 3x - x^2 - xy = b \\ \frac{dy}{dt} &= y(4-2x-y) = 4y - 2xy - y^2 = s\end{aligned}$$

find the Jacobian and evaluate it at  $(x, y) = (1, 2)$ .

$$\frac{\partial b}{\partial x} = 3 - 2x - y \quad \frac{\partial b}{\partial y} = -x$$

$$\frac{\partial s}{\partial x} = -2y \quad \frac{\partial s}{\partial y} = 4 - 2x - 2y$$

2 pts

$$\text{Jacobian} = \begin{bmatrix} 3 - 2x - y & -x \\ -2y & 4 - 2x - 2y \end{bmatrix}$$

at  $(1, 2)$   $Jac =$

$$Jac = \begin{pmatrix} 3 - 2 \cdot 1 - 2 & -2 \\ -2 \cdot 2 & 4 - 2 \cdot 1 - 2 \cdot 2 \end{pmatrix} \quad \text{Jacobian evaluated at } (1, 2) = \begin{bmatrix} -1 & -2 \\ -4 & -2 \end{bmatrix}$$

1 pt

For the matrix  $J = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$  find the characteristic equation and the eigenvalues.

Characteristic equation  $\lambda^2 - \lambda - 6$  1 pt

Eigenvalues  $3, -2$  1 pt

$$\text{Char. eqn} = \begin{vmatrix} \lambda - 2 & -1 \\ -4 & \lambda + 1 \end{vmatrix} = 0$$

$$= (\lambda - 2)(\lambda + 1) - (-1)(-4)$$

$$= \lambda^2 - \lambda - 2 - 4$$

$$= \lambda^2 - \lambda - 6$$

$$= (\lambda - 3)(\lambda + 2) = 0$$

$$\Rightarrow \lambda = 3, -2$$