Test 3

Name:_____

Show your work! Answers that do not have a justification will receive no credit.

1. (15 points) Compute the following integrals (if an integral diverges say so):

   (a) $\int_{-\infty}^{0} x^2 e^{2x}\, dx.$

   (b) $\int_{0}^{2} \frac{d\theta}{2-\theta}$

   (c) $\int_{-1}^{2} \frac{2\, dv}{v}$

2. (10 points) Find the sums of the following series

   (a) $1000 + 1000(1.05) + 1000(1.05)^2 + \cdots + 1000(1.05)^{29}$

   (b) $\sum_{k=3}^{\infty} \frac{3}{4^k}$

3. (15 points) For what values of x to the following series converge

(a) $\displaystyle\sum_{n=0}^{\infty} x^n 4^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n(x-2)^n}{3^n}$

4. (10 points) Find the third order Taylor polynomial for the function $f(x) = \sqrt{x}$ at the point $x = 9$.

5. (10 points) Let $f(x)$ be a function so that $f(1) = -1$, $f'(1) = 3$ and $f'(1) = -2$.

   (a) Draw a graph of $y = f(x)$ near $x = 1$.

   (b) Give the best approximation you can for $f(1.01)$.

6. (10 points) Compute the following

   (a) $\lim\limits_{x \to 0} \dfrac{\sin(2x)}{x}$

   (b) $\lim\limits_{x \to \infty} \dfrac{x^{100}}{(1.001)^x}$

7. (20 points) The probability that a light bulb burns out during its first t weeks of use has the probability density function

$$p(t) = \begin{cases} \frac{1}{40}e^{\frac{-t}{40}} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

(a) What is the probability that the a bulb lasts a year (52 weeks)?

(b) What is the cumulative distribution function?

(c) What is the median length of life of a light bulb?

(d) What is the mean length of life of a light bulb?

8. (10 points) How many terms of the series

$$1 - \frac{1}{2^3} + \frac{1}{3^3} - \frac{1}{4^3} + \frac{1}{5^3} - \frac{1}{6^3} + \cdots \pm \frac{1}{n^3} \cdots$$

do we need to be sure that we have the sum accurate to 6 decimal places?

Extra Credit (7 points) A ball is dropped straight down form a height of 10 feet and keeps bouncing so that each bounce is $\frac{3}{4}$ the height of the bounce before (so that the first bounce is 7.5 feet). What is the total distance the ball covers by the time it stops bouncing?