Test 1

Name:____________________________

Show your work! Answers that do not have a justification will receive no credit.

1. (25 points) Compute the following:

(a) $\displaystyle\int e^{4x}\sqrt{e^{4x}+1}\,dx$

(b) $\displaystyle\int (\theta^2 - 3\theta + 2)\sin(\theta/2)\,d\theta$

(c) $\displaystyle\int e^{ax}\sin(x)\,dx$ where a is a constant.

(d) $\displaystyle\int \frac{dx}{4+x^2}$

(e) $\displaystyle\int \frac{1+2t+3t^2}{t(1+t^2)}$

2. (20 points) Compute

(a) $\displaystyle\int_0^{\pi/4} \frac{d\alpha}{\sqrt{1-\alpha^2}}$

(b) $\displaystyle\int_0^{\pi} x\sin(nx)\,dx$ where n is a constant.

(c) The average value of the function $x\sin(x)$ on the interval $[0,\pi]$.

3. (10 points) Consider the region below $y = e^{-x/2}$, above the x-axis, to the right of $x = 0$, and to the left of $x = 2$. Compute the volume of the solid that is obtained by revolving this region around the x-axis.

4. (20 points) Solve the following initial value problems.

(a) $\dfrac{dA}{dt} = \dfrac{t}{A+2} \qquad A(1) = 4.$

(b) $y' = y(10 - y)$.

5. (15 points) A cool glass of lemonade, that starts at 40°F that is set out on a day when the temperature is 85°F. Assume that the lemonade warms up according to Newton's Law of Cooling which says that the rate of change of the temperature of the lemonade is proportional to the difference of temperature of the lemonade and the surrounding air temperature. Let t be the time in minutes since the lemonade was first set out. Assume that when $t = 20$ the temperature of the lemonade is 55°F.

(a) Write down the rate equation and the initial value satisfied by the temperature of the lemonade. Label all your variables and give their units. (This equation should involve a constant of proportionality coming from Newton's Law.)

(b) Solve this rate equation.

(c) What is the temperature of the lemonade after an hour?

6. (10 points) We know that the derivative of $\sec(x)$ is $\sec(x)\tan(x)$. (Or in other ward $(\sec(x))' = \sec(x)\tan(x)$. Use this to find a formula for the derivative of $\operatorname{arcsec}(x)$. (Recall arcsec is defined so that if $y = \operatorname{arcsec}(x)$ then $\sec(y) = x$.)