Review for Test 2

1. **Integration.** We have some now tricks for finding anti-derivatives since the last test. In particular we now know about substitution (§11.2 of the blue hand out) and integration by parts (§11.3 of the blue hand out). For practice you should look at the problems on pages 700 and 701 (for substitution) and page 706 problems 1abcdg, 3. We have also done some new applications of the integral. In particular now know how to use the integral to compute volumes and length. If the area of cross section of a solid at height y is $A(y)$ then the volume between the heights $y = a$ and $y = b$ is

$$\text{Volume} = \int_a^b A9y)\,dy$$

We also now know that the length of the graph $y = f(x)$ for $a \le x \le b$ is

$$\text{Length} = \int_a^b \sqrt{1 + f'(x)^2}\,dx.$$

2. **Vectors.** The the problems on the exam will follow very closely the ones on the last couple of worksheets. You should go over these in detail. Among the things you should know are

   (a) **Basic vector algebra.** Things like adding and subtracting vectors and graphing them. Given two points points P and Q be able to find the displacement vector $\overrightarrow{PQ}$.

   (b) **Inner products.** You should be able to use the inner product (which is anther name for the scalar product) to compute the length of vectors and the angles between vectors. The basic formulas are

$$\mathbf{v} \cdot \mathbf{v} = |\mathbf{v}|^2, \qquad \mathbf{v} \cdot \mathbf{w} = |\mathbf{v}||\mathbf{w}|\cos(\theta).$$

   where θ is the angle between $\mathbf{v}$ and $\mathbf{w}$. Closely related to the inner product is the "work" which is defined to be the inner product of force $\mathbf{F}$ and displacement $\mathbf{D}$. That is

$$\text{Work} = \mathbf{F} \cdot \mathbf{D}$$

   (c) **Vector form of a line.** If $\mathbf{u}$ and $\mathbf{v}$ are vectors then $\mathbf{r}(t) = \mathbf{u} + t\mathbf{v}$ is a straight line through $\mathbf{u}$ and in the direction $\mathbf{v}$.

3. **Curves.** We have three ways of giving a curve. The first is as on old fashioned equation in x and y. Then next is as parametric equations where x and y are given as functions of t. As example would be $x(t) = e^t$, $y(t) = e^{-t}$ which is a parametric equation of hyperbolic $xy = 1$. Lastly we can give the curve in vector form. That is $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$. For a curve in vector form we can compute the velocity $\mathbf{v}(t)$, the acceleration $\mathbf{a}(t)$ and the speed. These are given by

$$\begin{aligned} \mathbf{v}(t) &= \mathbf{r}'(t) \\ \mathbf{a}(t) &= \mathbf{v}'(t) = \mathbf{r}''(t) \\ \text{speed} &= |v(t)|. \end{aligned}$$

We have also shown that the length of a curve in vector form is

$$\text{Length;} \int_a^b |\mathbf{r}'(t)|\,dt = \int_a^b \sqrt{x'(t)^2 + y'(t)^2}\,dt$$

4. **Transformations.** We might not get to this in time for the test, but I am going to try. For this look at the handout A Brief Introduction to Vectors and work the problems on worksheet 3.