Homework

Some new Derivative Formulas
Due Monday

This is to go along with the computer related problems you have already been given. You will be ask to use the chain rule to derive some new derivative formulas. Let $y = y(x)$ be a function of x. This for any function $f(y)$ the chain rule implies

$$\frac{d}{dx}f(y) = f'(y)y'.$$

Thus if $f(y)$ happens to be the *inverse function* of y, that is $f(y) = x$, or to show the dependence on x, $f(y(x)) = x$, then from the above

$$f'(y)y' = \frac{d}{dx}f(y) = \frac{d}{dx}x = 1$$

and we can solve for y' to get $y' = \dfrac{1}{f'(y)} = \dfrac{1}{f'(y(x))}$. Therefore if we know the derivative of the function f we can use this to find the derivative of the inverse function $f(y)$.

You have already seen an example of this when we derived the derivative of ln. Let $y = \ln(x)$. Then we want to find a formula for $y' = (\ln(x))'$. From the definition of$\ln(x)$ we

$$y = \ln(x) \qquad \text{implies} \qquad e^y = x.$$

Now take the derivative of $e^y = x$ with respect to x and use the chain rule

$$\frac{d}{dx}e^y = e^y y' = \frac{d}{dx}x = 1$$

Now solve for y' to get

$$y' = \frac{1}{e^y} = \frac{1}{e^{\ln(x)}} = \frac{1}{x},$$

where we have used $y = \ln(x)$ and $e^{\ln(x)} = x$. But as $y = \ln(x)$ this shows $(\ln(x))' = \frac{1}{x}$.

Here are some for you to try.

1. Recall that $\arcsin(x)$ is the function to that $\sin(\arcsin(x)) = x$. Thus if $y = \arcsin(x)$ then $\sin(y) = x$. Take the derivative of this and use the result to derive a formula for the derivative of $\arcsin(x)$.

2. The function $\arctan(x)$ is the function so that $\tan(\arctan(x)) = x$. So we can again set $y = \arctan(x)$. Then $\tan(y) = x$. As above take the derivative of this and derive a formula for the derivative of $\arctan(x)$.