Integration

By the fundamental theorem of calculus we now know that being able to find anti-derivatives is very useful in being able to evaluate integrals and thus in computing things like areas, volumes, lengths etc. We will use the integral sign without limits to denote an anti-derivatives. That is

$$\int f(x)\,dx = \text{The anti-derivative of } f(x).$$

By working backwards from the integrals we know we can easily make a table of basic anti-derivatives. In the following a is a constant.

$$\begin{aligned}
\int x^p\,dx &= \frac{1}{p+1}x^{p+1} + C \\
\int \frac{1}{x}\,dx &= \ln|x| + C \\
\int \sin(ax)\,dx &= -\frac{1}{a}\cos(ax) + C \\
\int \cos(ax)\,dx &= \frac{1}{a}\sin(ax) + C \\
\int e^{ax}\,dx &= \frac{1}{a}e^{ax} + C \\
\int b^x\,dx &= \frac{1}{\ln b}b^x + C
\end{aligned}$$

These you should know from memory (but I hope that by this point we have done enough examples that are more or less "obvious"). We can use some of the derivative formulas that we derived for the inverse trig functions to give some farther formulas that will come up fairly often

$$\begin{aligned}
\int \frac{dx}{a^2+x^2} &= \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C \\
\int \frac{dx}{\sqrt{a^2-x^2}} &= \arcsin\left(\frac{x}{a}\right) + C
\end{aligned}$$

Latter in the term more formulas will come up, but this is more that enough for the time being. There are some basic rules for integration. The most basic is linearity

$$\int (af(x)+bg(x))\,dx = a\int f(x)\,dx + b\int g(x)\,dx.$$

Next comes the change of variable formula, also called the method of substitution. For this read §11.2 pages 679–685 of Vol. II.

Here are some homework problems due next week (these are from Vol. II).

1. Page 674 numbers 3, 5.
2. Pages 682–683 1, 2, 3abc