Mathematics 141 Final

Name:______________________________

Show your work to get credit. An answer with no work will not get credit.

(1) (30 points) Compute the following derivatives. You do not have to simplify your answers.

(a) $y = 5x^3 - 7x^2 + 4x - 6$

$y' =$

(b) $y = \dfrac{3}{x^5} + \dfrac{2}{\pi^3}$

$y' =$

(c) $A(t) = 4\sqrt[3]{t} - \dfrac{7}{\sqrt{t}}$

$A'(t) =$

(d) $y = \sin(x)$

$y' =$

(e) $y = \cot(3x)$

$y' =$

(f) $y = \sec(5x)$

$y' =$

(g) $P(t) = 3t^2 \sin(t)$

$P'(t) =$

(h) $R(t) = \dfrac{1 + \sin(t)}{1 - \sin(t)}$

$R'(t) =$

(i) $y = 2\sin^4(3x - 1)$

$y' =$

(j) $R(t) = 5(3t^2 + 2t + 1)^{11}$

$R'(t) =$

(k) $A(\theta) = \dfrac{3}{\sqrt{4 - \sin(2\theta)}}$

$A'(\theta) =$

(l) $G(x) = \displaystyle\int_1^x \cos(t^2)\, dt$

$G'(x) =$

(m) $F(x) = \int_1^{x^2} \sin(t^3)\,dt$

$F'(x) =$

(2) (30 points) Compute the following antiderivatives.

(a) $\int (3x^4 - 6x^3 + 4x^2 - 5)\,dx$ ____________________

(b) $\int \left(\frac{2}{s^3} - \frac{2}{\pi^3}\right) ds$ ____________________

(c) $\int \frac{3t^2 + 4t + 1}{\sqrt{t}}\,dt$ ____________________

(d) $\int (3\cos t - 4\sin t)\,dt$ ____________________

(e) $\int (\cos(3\theta) - \sin(4\theta))\,d\theta$ ____________________

(f) $\displaystyle\int \frac{x^3}{\sqrt{x^4+1}}\,dx$ ____________________

(g) $\displaystyle\int 2\sin^3(4t)\cos(4t)\,dt$ ____________________

(h) $\displaystyle\int \frac{y^3+1}{(y^4+4y+2)^5}\,dy$ ____________________

(3) (15 points) Compute the following definite integrals.

(a) $\displaystyle\int_{-1}^{2} (6x^2-8x+1)\,dx$ ____________________

(b) $\displaystyle\int_0^{\pi/2} \sin 3t\, dt$ ____________________

(c) $\displaystyle\int_{-1}^{1} \frac{x^2}{(x^3+1)^2}\, dx$ ____________________

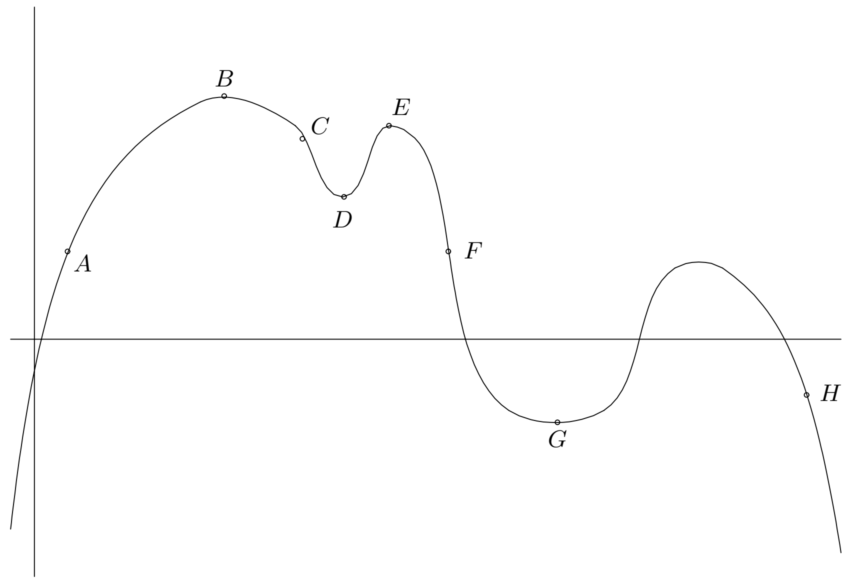
(d) $\displaystyle\int_3^5 3x\sqrt{25-x^2}\, dx$ ____________________

(4) (5 points)

(a) State the mean value theorem.

(b) Show that if $a, b \geq 1$, then $|\sqrt{b} - \sqrt{a}| \leq \frac{1}{2}|b-a|$.

(5) (5 points) Let $y = f(x)$ have the following graph.



(a) At which points is $f'(x) > 0$? ________________

(b) At which points is $f'(x) = 0$? ________________

(c) At which points is $f''(x) < 0$? ________________

(d) At which points does f have a local maximum? ________________

(e) At which points does f have a local minimum? ________________

(6) (5 points) Compute the first three derivatives of $f(x) = x \sin(x)$.

$f'(x) =$

$f''(x) =$

$f'''(x) =$

(7) (5 points) If x and y are related by

$$x^2 + 2xy - 4y^2 + 2x - 4y = 4$$

then find $\frac{dy}{dx}$ by implicit differentiation.

$\frac{dy}{dx} =$ ________________

(8) (5 points) Find the tangent line to $2x^2 + 2xy + y^2 = 10$ at the point $(1, 2)$.

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(9) (5 points) Solve $\dfrac{dy}{dx} = \dfrac{x^2}{y^3}$ with $y(1) = 2$. ____________________

(10) (8 points) A 10 foot long ladder is leaning against the wall of a stage, but the base is slipping away from the wall at 2 ft/sec. How fast is the top of the ladder moving when it is 6 feet from the ground?

Rate top is moving = ____________________

(11) (5 points) For the function $f(x) = 2x^3 - 24x + 1$ on $[-4, 3]$, sketch a graph showing the critical points, the local maximums, the loical minimums, and the inflection points.

(12) (7 points) A farmer has 100 feet of fencing and wishes to make an enclosure with two pens and one side along along a barn as shown in figure 1. What are the dimensions that give the largest total area for the pens?

Length = ____________________

Width = ____________________

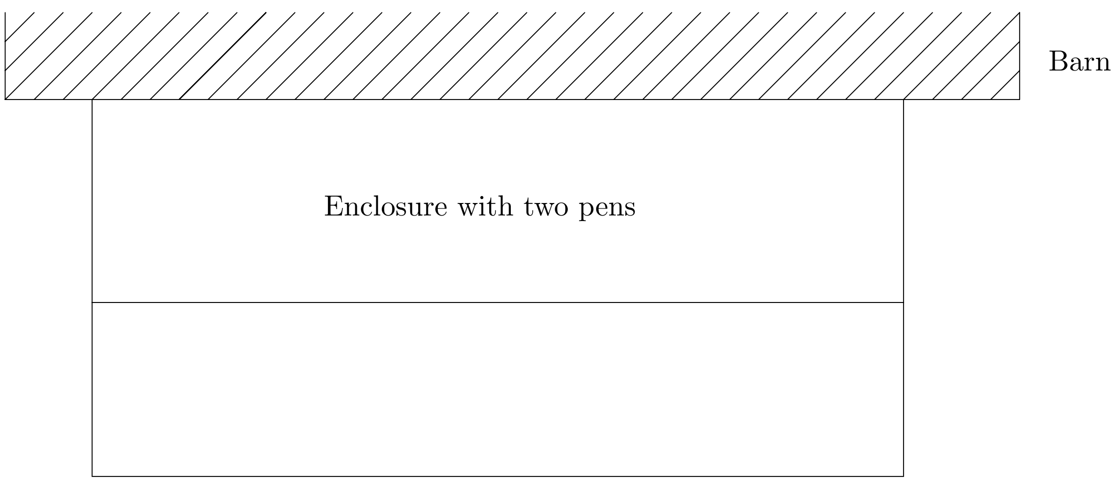

FIGURE 1. One side is along the barn.

(13) (5 points) If $f'(x) = 3x^2 + 4x$ and $f(2) = 3$, then find $f(x)$.

$f(x) =$ ____________________

(14) (10 points) Let $y = f(x)$ be a function on $[0, 10]$ with the properties

- f' on the intervals $(0, 2)$ and $(7, 10)$.
- $f' < 0$ on the interval $(2, 7)$.
- $f'' < 0$ on the interval $(0, 5)$
- $f'' > 0$ on the interval $(5, 10)$
- $f(0) = 6$, $f(2) = 8$, $f(7) = 2$, and $f(10) = 5$.

(a) Sketch a graph of $y = f(x)$ on the interval $[0, 10]$.

(b) What what is the maximum value of $f(x)$? ________________

(c) What is the value of x that minimizes $f(x)$ ________________

(d) At what x value does $f(x)$ have an inflection point? ________________

(e) What are the stationary points of $f(x)$ ________________

(15) (5 points) Graph both $y = x^2 + 2$ and $y = 4 - x$ on the same graph showing the points of intersection of the curves. Then find the area bounded between them.

Area = ________________

Graph:

(16) (5 points) What is the volume when the region bounded by the lines $y = 0$, $y = \sqrt{x}$ and $x = 4$ is revolved about the x-axis?

Volume = ______________________

(17) (5 points) Set up the integral to find the arclenth of the curve $y = x^3$ between $x = 1$ and $x = 4$.

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Have nice holiday.