

Mathematics 141 Test #2

Name: _____

Show your work to get credit. An answer with no work will not get credit.

(1) (25 points) Compute the following (knowing what the notation means is part of the problem).

(a) $\frac{dy}{dx}$ where $y = \tan(x^2)$ $\frac{dy}{dx} =$ _____

(b) $\frac{ds}{dt}$ where $s = 5(t^2 + t + 1)^{50}$. $\frac{ds}{dt} =$ _____

(c) $f''(x)$ where $f(x) = 4x^3 + 7x^2 - 2x + 1$ $f''(x) =$ _____

(d) $\frac{d}{dt}(t \sin^2(t) + t^2)$ _____

(e) $\frac{d^2y}{dx^2}$ where $y = \frac{4}{x^3}$ $\frac{d^2y}{dx^2} =$ _____

(f) $\frac{ds}{dt}$ where $s = \sqrt[3]{1 + \tan(t)}$ $\frac{ds}{dt} =$ _____

(g) (5 points) $\frac{dA}{dr}$ where $A = \frac{4}{\sqrt{r^3 + r}}$ $\frac{dA}{dr} =$ _____

(2) (5 points) If u and v are related by $u^2 + 4uv + 2v^2 = 10$ find $\frac{dv}{du}$ by implicit differentiation.

$\frac{dv}{du} =$ _____

(3) (5 points) Find the tangent line to $x^2y + 2xy^2 = 10$ at the point $(1, 2)$.

(4) (10 points) A 20 foot long ladder is leaning against the side of a building, but the base is slipping away from the building at 3 ft/sec. How fast is the top of the ladder moving when it is 12 feet from the ground?

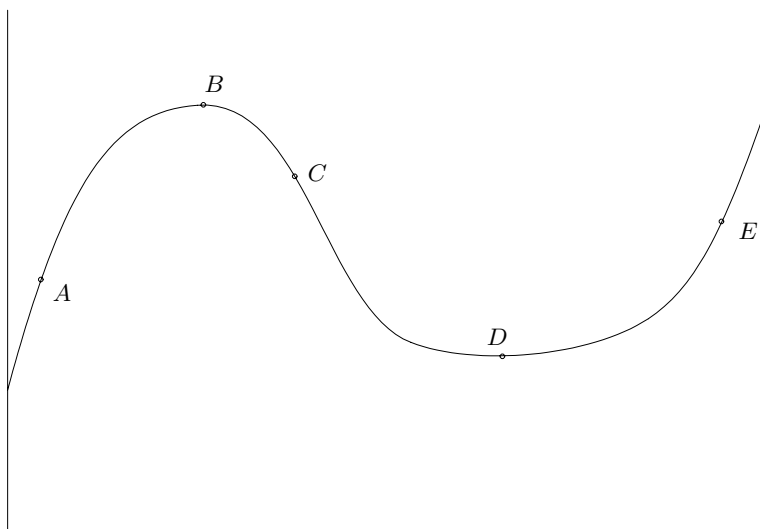
Rate top is moving = _____

(5) (10 points) Draw graphs of functions $f(x)$ with the following properties.

(a) $f'(x) < 0$ and $f''(x) > 0$.

(b) $f(2) = 3$, $f'(2) = 0$, and $f''(x) < 0$.

(6) (10 points) In the following figure



(a) At which of the labeled points is $f' > 0$?

(b) At which of the labeled points is $f' = 0$?

(c) At which of the labeled points is $f'' > 0$?

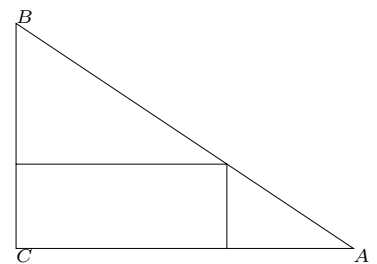
(d) At which of the labeled points is $f'' < 0$?

(7) (10 points) Find the maximum and minimum of $y = 3x^2 - x^3$ on the interval $[0, 4]$.

(8) (15 points) Sketch the graph, labeling all the local maximums, local minimums and inflection points of a function $y = f(x)$ on $[1, 4]$ with the following properties:

- $f' > 0$ on the intervals $(1, 2)$ and $(3, 4)$,
- $f' < 0$ on the interval $(2, 3)$,
- $f'' < 0$ on $(1, 2.5)$,
- $f'' > 0$ on $(2.5, 4)$, and
- $f(1) = 3$, $f(2) = 6$, $f(3) = 5$, $f(4) = 9$.

(9) (10 points) The right triangle $\triangle ABC$ has side AC of length 6 and side BC of length 3. What are the lengths of the sides of the rectangle of largest area that can be inscribed in $\triangle ABC$ as in the figure?



(10) 5 free points.