

### Test 3

Name: \_\_\_\_\_

**Show your work!** Answers that do not have a justification will receive no credit.

1. (30 Points) Take the derivatives of the following functions. In the first blank put the unsimplified form. If there is a second blank then put in the simplified form of the derivative.

(a)  $f(x) = \sin(x) + e^x$

$$f'(x) = \underline{\hspace{10cm}}$$

(b)  $w = e^{\theta^2+3\theta}$

$$\frac{dw}{d\theta} = \underline{\hspace{10cm}}$$

(c)  $f(x, y) = x^3y^2 + 2x + 9y^4$

$$\frac{\partial f}{\partial x} = \underline{\hspace{10cm}}$$

$$\frac{\partial f}{\partial y} = \underline{\hspace{10cm}}$$

(d)  $Q(u, v) = e^u \sin(u + v)(u^2 + v^2)$

$$\frac{\partial Q}{\partial u} = \underline{\hspace{10cm}}$$

$$\frac{\partial Q}{\partial v} = \underline{\hspace{10cm}}$$

$$(e) \ w = \frac{x^2 + y^2}{x^2 - y^2}$$

$$\frac{\partial w}{\partial x} = \underline{\hspace{15em}}$$

$$= \underline{\hspace{15em}}$$

Simplified form

$$\frac{\partial w}{\partial y} = \underline{\hspace{15em}}$$

$$= \underline{\hspace{15em}}$$

Simplified form

2. (10 Points) Let  $f(x, y) = \sqrt{x + y^2}$ . What is the full microscope equation for  $f$  at the point where  $x = 5$  and  $y = 2$ ?

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3. (10 Points) Let  $V(u, v)$  be a function so that  $V(2, -3) = 4$ ,  $\frac{\partial V}{\partial u}(2, -3) = 1.5$ , and  $\frac{\partial V}{\partial v}(2, -3) = 2.5$ . Then

(a) What is a good estimate of  $V(2.3, -3.1)$ ?

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(b) What is a good estimate for the solution to  $V(u, -2.9) = 3.8$ ?

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4. (20 Points) Recall that NEWTON'S LAW OF COOLING states that a object cools at a rate proportional to the difference in its temperature with the temperature of the surrounding air. Assume that the air temperature this morning was  $50^{\circ}\text{F}$  and that after driving in the engine of my car was  $135^{\circ}\text{F}$  and that after standing in the parking lot for 45 minutes that the temperature of the engine was  $105^{\circ}\text{F}$ .

(a) What is temperature of the engine  $t$  minutes after I parked the car? (Label all variables and show all work.)

(b) How long will it take the engine to cool down to  $70^{\circ}\text{F}$ ?

(c) Assume that the air temperature stays at  $50^{\circ}$  what is the temperature of the engine when I start it up after have letting it set for 8 hours?

5. (10 Points) Solve the initial value problem  $y' = 4y$ ,  $y(0) = -7$ .

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6. (15 Points) Assume that 5 fleas make their home on a previously flea-less dog, and that if  $F(t)$  is the number of fleas on the dog after  $t$  days that  $F(t)$  grows by the logistic equation.

$$F'(t) = .1F(t) \left(1 - \frac{F(t)}{1200}\right)$$

(a) What is the microscope equation for  $F(t)$  at  $t = 0$ ?

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(b) Give an estimate for the number of fleas after one day.

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(c) At some point we expect the size of the population of the number of fleas to level out and stay constant. What is the size of the flea population that this occurs? Explain your answer.

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7. (5 Points) Is  $y = x^3 + 1$  a solution to the initial value problem

$$3xy' = y - 1, \quad y(0) = 1?$$

Explain in a couple of English sentences why  $y$  is or is not a solution.