Final Name: _____

Show your work! Answers that do not have a justification will receive no credit.

Circle your answers.

1. (30 points) Find the derivatives of the following functions. If it is function of more than one variable, then find all the partial derivatives. You do not have to simplify your answers.

(a) $4x^7 + 3x + \sqrt[3]{x+4}$.

(b) $\dfrac{1-\sin(x)}{1+\cos(x)}$

(c) $\ln(x^3 + 4x)$

(d) $\dfrac{x^4}{x^4+3}$

(e) $e^{\tan(2\theta)}$

(f) $\cos(\theta^2 + 1)\sec(\theta^2 + 1)$

(g) $\sqrt{3y + e^x}$

(h) 17^{2u+5}.

(i) $x^3(x^4 + 1)(x^5 + 2)$

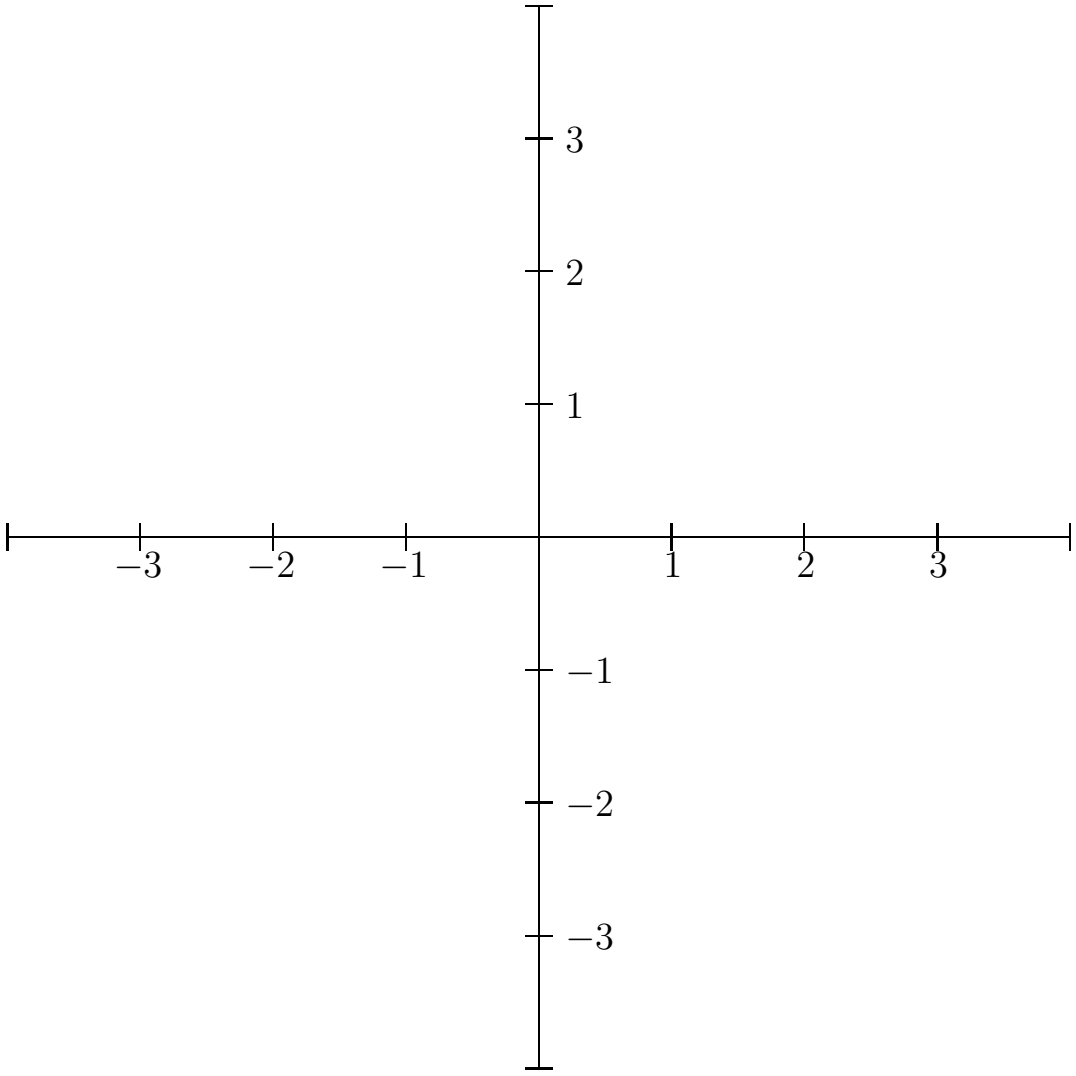
(j) $\dfrac{x^2 - y^2}{x^2 + y^2}$.

2. (15 points) (a) Write the microscope equation for $y = 3x^2 + x$ at the point where $x = -1$.

(b) What is the equation of the tangent line to $y = 3x^2 + x$ at the point where $x = -1$?

(c) Write the full microscope equation for $V = \sqrt{2u^2 + v}$ at the point where $(u, v) = (2, 1)$.

3. (10 points) Sketch the graph the solution to the initial value problem $y' = \dfrac{(t+1)(t-2)}{4+y^2}, \quad y(1) = 0$ and label all the local maxima and minima.



4. (15 points) At 12:00 noon we add 3 grams of yeast to a large vat of grape juice. After two hours there are 6 grams of yeast in the vat. Assume that the rate of growth of the yeast is proportional to the amount of yeast present.

(a) Write a rate equation and an initial value for the growth of the yeast in the vat as a function of time. Label all variables and give their units.

(b) Find a formula for the amount of yeast present in the vat t hours after the yeast was added.

(c) How long until there are 100 grams of yeast in the vat?

5. (10 points) A truck driver starts a trip by driving at 50mph for 2 hours and then speeds up to 60 mph for 3 hours. She then stops for lunch which takes an hour and resumes the trip by driving at 40 mph for anther 5 hours.

(a) Graph the speed of the truck as a function of the time t in hours form the start of the trip. (draw and label your own axis.)

(b) Find formulas for the distance $D(t)$ covered for t in the intervals (i) $2 < t < 5$ and (ii) $6 < t < 11$.

(c) How long (starting from the beginning of the trip) does it take for the truck to cover 400 miles?

6. (15 points) Measurements are made of the length L (measured in cm) of a brass rod at different temperatures . Some the information involved is given in the table at the right (measured in °F).

T	L
56	188.79
58	188.90
60	189.01
62	189.12
64	189.23

(a) Give an estimate for the rate of change of L with respect to T when $T = 60°\text{F}$.

(b) Write the microscope equation at the point where $T = 60°\text{F}$.

(c) Estimate the temperature at which the length is 188.95

7. (10 points) Fill in the blanks.

(a) If $f(3,-2)=4$, $\frac{\partial f}{\partial x}(3,-2)=2$ and $\frac{\partial f}{\partial y}(3,-2)=-3$, then a reasonable estimate of $f(3.2,-2.1)$ is _____.

(b) If $g(2,3)=1$, $\frac{\partial g}{\partial x}(2,3)=2$, $\frac{\partial g}{\partial y}(2,3)=-3$, then a reasonable estimate of the solution to $g(1.8,y)=0$ is _____.

(c) If $h(3,2)=1$, $h(3.1,1.9)=1.3$ and $\frac{\partial h}{\partial y}(3,2)=-2$, then a reasonable estimate of $\frac{\partial h}{\partial x}(3,2)$ is _____.

8. (10 points) (a) If $f'(t)=0$ and $A(6)=4$ then find $f(t)$

(b) If $y'(t)=5y(t)$ and $y(0)=2$ then find $y(t)$.

(c) If $H'(s) = 3s^5 - 9s^2$ and $H(0) = 3$ then what is $H(s)$?

9. (15 points) Let $P(t)$ be a solution to the initial value problem $P'(t) = .1P(t)(10 - P(t))$, $P(0) = 1$.
(a) Compute $P'(0)$

(b) Estimate $P'(1)$ by taking two steps in Euler's method.

10. (5 points) Is $y = 2x$ a solution to the initial value problem $y' = xy$, $y(1) = 2$? Explain you answer in a couple of sentences.

11. (15 points) The volume of a cylinder with height h and circler base r is $V + \pi r^2 h$.
(a) If r is increased by 20% then what is the percentage increase in V?

(b) Write the total microscope equation for ΔV when $r = 3$ and $h = 10$.

(c) If we measure and find that the radius is $r = 3$ with an error of $\pm.1$ and the height is $h = 10$ with an error of $\pm.2$ then approximate the error in using $\pi 3^3 \cdot 10 \approx 282.7433\ldots$ as the volume of the cylinder.