Final

Name: _____

Show your work! Answers that do not have a justification will receive no credit.
Circle your answers.

1. (30 points) Find the derivatives of the following functions. If it is function of more than one variable, then find all the partial derivatives. You do not have to simplify your answers.

(a) $9x^3 + \sqrt[3]{x+1}$.

(b) $\dfrac{1-\cos(x)}{1+\cos(x)}$

(c) $\ln(x^4 + 3x)$

(d) $\dfrac{\ln(xy^2)}{y}$

(e) $e^{\tan(2\theta)}$

(f) $\cos(\theta^2+1)\sec(\theta^2+1)$

(g) $\sqrt{x+e^y}$

(h) 13^{4u-9}.

(i) $u^4 \cot(v + 2w)$.

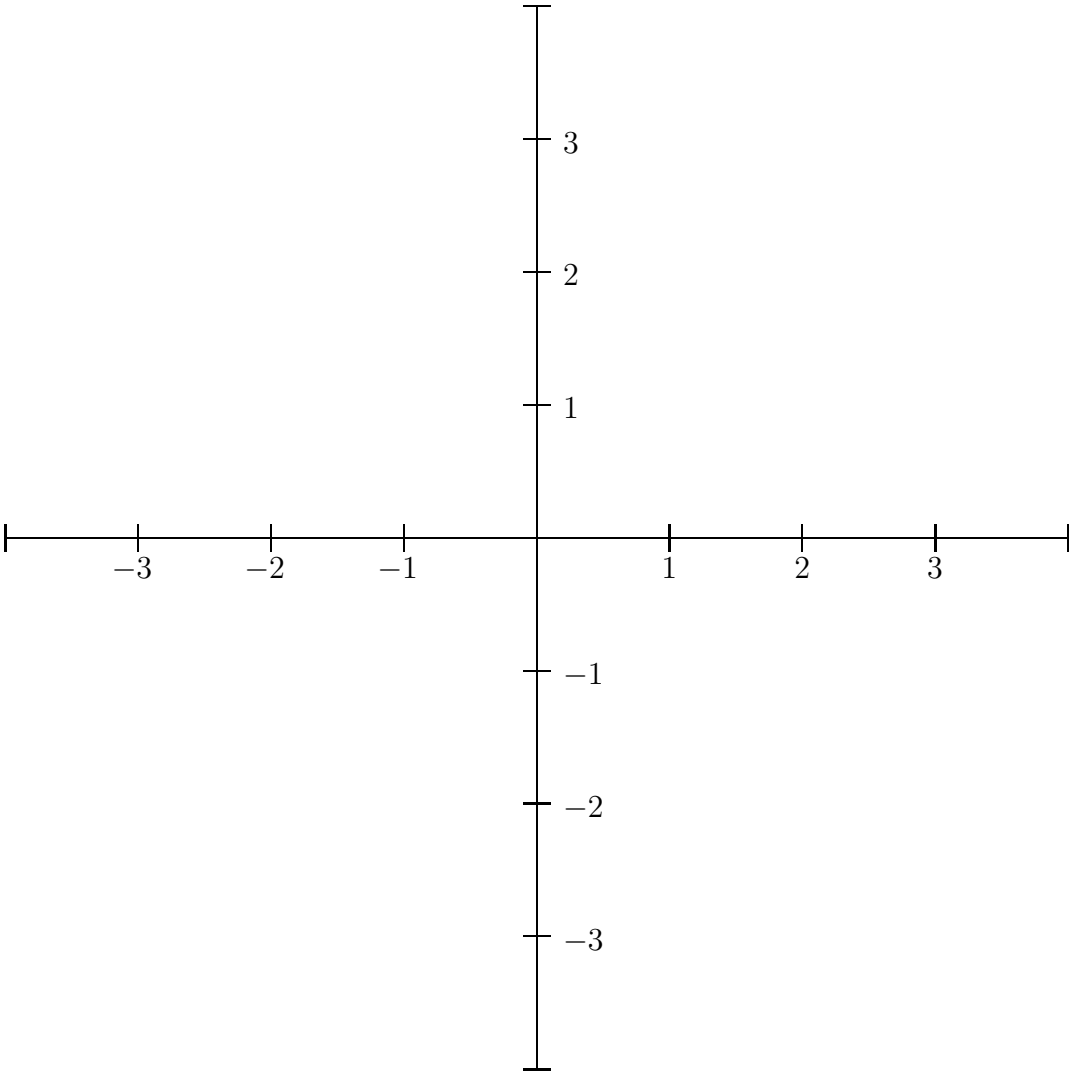
(j) $\sqrt{\ln(t^2 + 5)}$.

2. (10 points) (a) Write the microscope equation for $y = 3x - 2x^2$ at the point where $x = -1$.

(b) What is the equation of the tangent line to $y = 3x - 2x^2$ at the point where $x = -1$?

(c) Write the full microscope equation for $V = \sqrt{2u^2 + v}$ at the point where $(u, v) = (2, 1)$.

3. (10 points) Sketch the graph the solution to the initial value problem $y' = \dfrac{(t-1)(t+2)}{4+y^2}$, $y(2) = 0$ and label all the local maxima and minima.



4. (15 points) At 12:00 noon we add 2 grams of yeast to a large vat of grape juice. After three hours there are 4 grams of yeast in the vat. Assume that the rate of growth of the yeast is proportional to the amount of yeast present.
(a) Write a rate equation and an initial value for the growth of the yeast in the vat as a function of time. Label all variables and give their units.

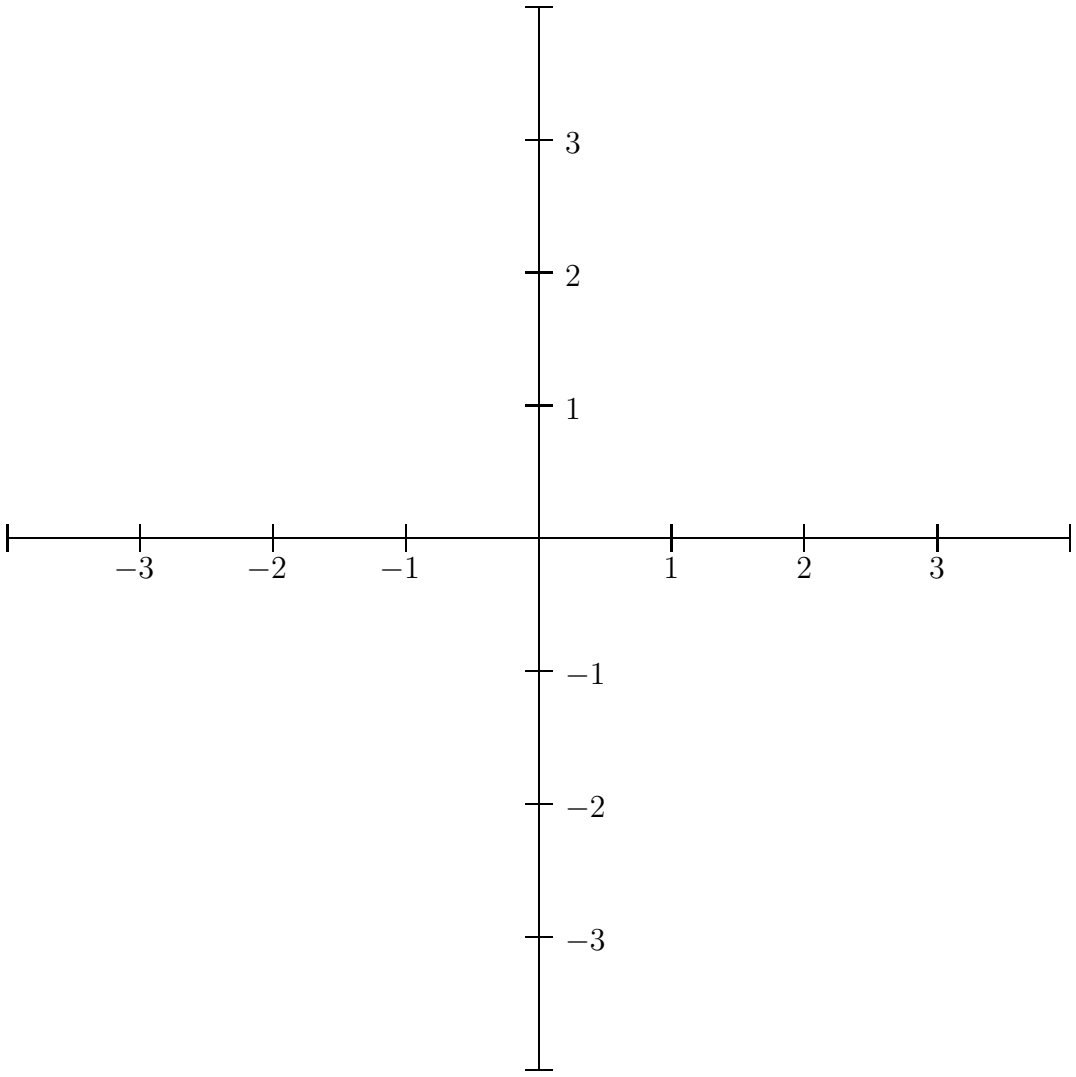
(b) Find a formula for the amount of yeast present in the vat t hours after the yeast was added.

(c) How long until there are 100 grams of yeast in the vat?

5. (15 points) Let $y(t)$ be defined by the initial value problem

$$y' = .5(y+1)(y-2), \qquad y(-1) = 1.$$

(a) Graph y on the interval $-3 \leq t \leq 3$.



(b) Estimate both $y(-326.7)$ and $y(439.5)$.

(c) What is the microscope equation for y at the point where $t = -1$?

(d) Estimate $y(-.95)$.

5. (15 points) Measurements are made of the length L (measured in cm) of a brass rod at different temperatures . Some the information involved is given in the table at the right (measured in °F).

T	L
56	188.79
58	188.90
60	189.01
62	189.12
64	189.23

(a) Give an estimate for the rate of change of L with respect to T when $T = 60°\text{F}$.

(b) Write the microscope equation at the point where $T = 60°\text{F}$.

(c) Estimate the temperature at which the length is 188.95

7. (10 points) Fill in the blanks.

(a) If $f(3,-2) = 4$, $\dfrac{\partial f}{\partial x}(3,-2) = 2$ and $\dfrac{\partial f}{\partial y}(3,-2) = -3$, then a reasonable estimate of $f(3.2,-2.1)$ is _____.

(b) If $g(2,3) = 1$, $\dfrac{\partial g}{\partial x}(2,3) = 2$, $\dfrac{\partial g}{\partial y}(2,3) = -3$, then a reasonable estimate of the solution to $g(1.8,y) = 0$ is _____.

(c) If $h(3,2) = 1$, $h(3.1, 1.9) = 1.3$ and $\frac{\partial h}{\partial y}(3,2) = -2$, then a reasonable estimate of $\frac{\partial h}{\partial x}(3,2)$ is _____.

8. (10 points) (a) If $A'(t) = 0$ and $A(-2) = 5$ then find $A(t)$

(b) If $y'(t) = 6y(t)$ and $y(0) = 4$ then find $y(t)$.

(c) If $H'(s) = 3s^4 - 9s^2 - sqrts$ and $H(1) = 3$ then what is $H(s)$?

9. (15 points) Of all rectangles with perimeter of length 60, which one has the largest area?

10. (5 points) Find the value of the constant a so that $u(t) = 4e^{-t/2} + a$ is a solution to

$$u' = -\frac{1}{2}(u - 45).$$

11. (15 points) The volume of a cylinder with height h and circler base r is $V + \pi r^2 h$.
(a) If r is increased by 20% then what is the percentage increase in V?

(b) Write the total microscope equation for ΔV when $r = 3$ and $h = 10$.

(c) If we measure and find that the radius is $r = 3$ with an error of $\pm.1$ and the height is $h = 10$ with an error of $\pm.2$ then approximate the error in using $\pi 3^3 \cdot 10 \approx 282.7433\ldots$ as the volume of the cylinder.