

(1) Let a and b be constants.

(a) $P(t) = 4e^t + 9\ln(t) + \frac{3}{t^4}$
 $= 4e^t + 9\ln(t) + 3t^{-4}$

$P'(t) = 4e^t + \frac{9}{t} - 12t^{-5}$

(b) $w = 3ab^3z^4 + b^2a^x + \underbrace{9a^{17}b^9}_{\text{constant}}$
 so its derivative = 0

$\frac{dw}{dz} = 12ab^3z^3 + b^2\ln(a)a^x$

(2) Let $f(x) = \sqrt{x}$.

(a) Find the equation of the tangent line to $y = f(x)$ where $x = 4$. (Note this asks for an equation, thus if there is no equal sign in your answer, it is wrong.)

$y = f(a) + f'(a)(x-a)$

$y = 2 + \frac{1}{4}(x-4)$

$f(x) = \sqrt{x} = x^{\frac{1}{2}}$

or $y = \frac{1}{4}x + 1$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$a = 4$

$f(a) = \sqrt{4} = 2$ $f'(a) = \frac{1}{2}(4)^{-\frac{1}{2}} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

(b) Use the tangent line to approximate $\sqrt{4.1}$.

$y = 2 + \frac{1}{4}(4.1-4)$

$\sqrt{4.1} \approx 2.025$

$= 2 + \frac{1}{4}(.1)$

$= 2.025$

(c) What is the value of $\sqrt{4.1}$ given by your calculator to 5 decimal places.

2.0248456731

Graph of $y = \sqrt{x}$ and its tangent line at $x = 4$ between $x = 3$ and $x = 5$ showing why the tangent line gives a good approximation to \sqrt{x} for x close to 4.

