Mathematics 122 Final Name:______________________________

You are to use your own calculator, no sharing.
Show your work to get credit.

(1) (10 Points) Corresponding values of S and t are given by the table:

S	2	7	12	17
t	20	17	14	11

(a) Explain why these values can come form a linear function.

(b) Find S as a linear function of t. ____________________

(c) What is the value of S when $t = 20$? ____________________

(2) (5 points) The following table comes from an exponential function.

t	0	1	2	3
P	5	10	20	40

(a) Write P as a function of t $P(t) =$ ____________________

(b) What is the value of P when $t = 1.5$? ____________________

(3) (10 points) The cost C of a pizza is proportional to the square of its diameter D. Assume that the cost of a 10 inch pizza is \$8.00.

(a) Give a formula for the cost of a pizza in terms of its diameter.

$C =$ ____________________

(b) What is the cost of an 18 inch pizza? ____________________

(4) (5 points) How long does it take a dollar invested at 8% interest, compounded monthly, to double?

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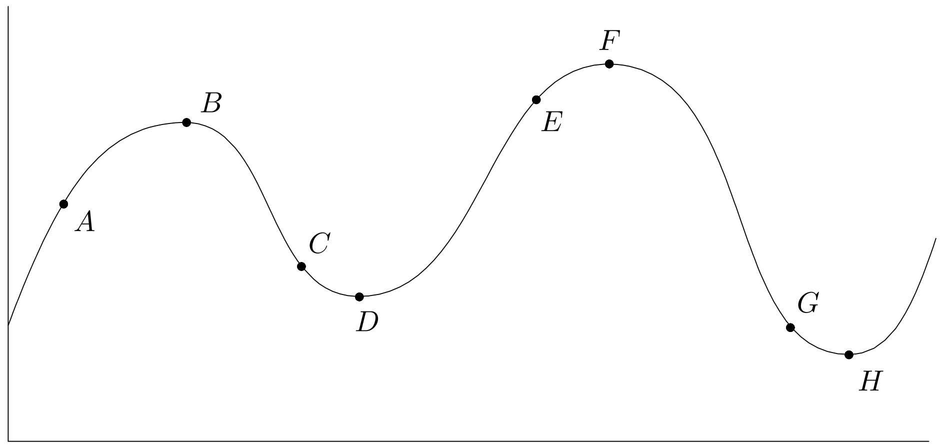
(5) (5 Points) Let $y = f(x)$ have the following graph.



(a) For which of the labeled points is $f'(x) > 0$? ______________________

(b) Which of the labeled points are critical points? ______________________

(c) For which of the labeled points if $f''(x) < 0$? ______________________

(d) Which of the labeled points are local maximums? ______________________

(6) (5 points) The weight w, in pounds, of a pine tree is a function of its height h in feet. That is $w = f(h)$. If the weight of a 50 foot is 817 pounds and $f'(50) = 45$ then estimate the height of a pine tree that is 52.3 feet tall.

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(7) (10 points) Let $f(x)$ have values as given in the following table.

x	0	2	4	6	8
$f(x)$	43	39	31	27	19

(a) Make a table of values of $f'(x)$.

(b) Estimate $f(2.3)$

$f'(2.3) \approx$ ____________________

(c) Make a table of values for $f''(x)$.

(8) (5 points) Find the equation of the tangent line to $y = 2x - x^2$ at the point where $x = 3$.

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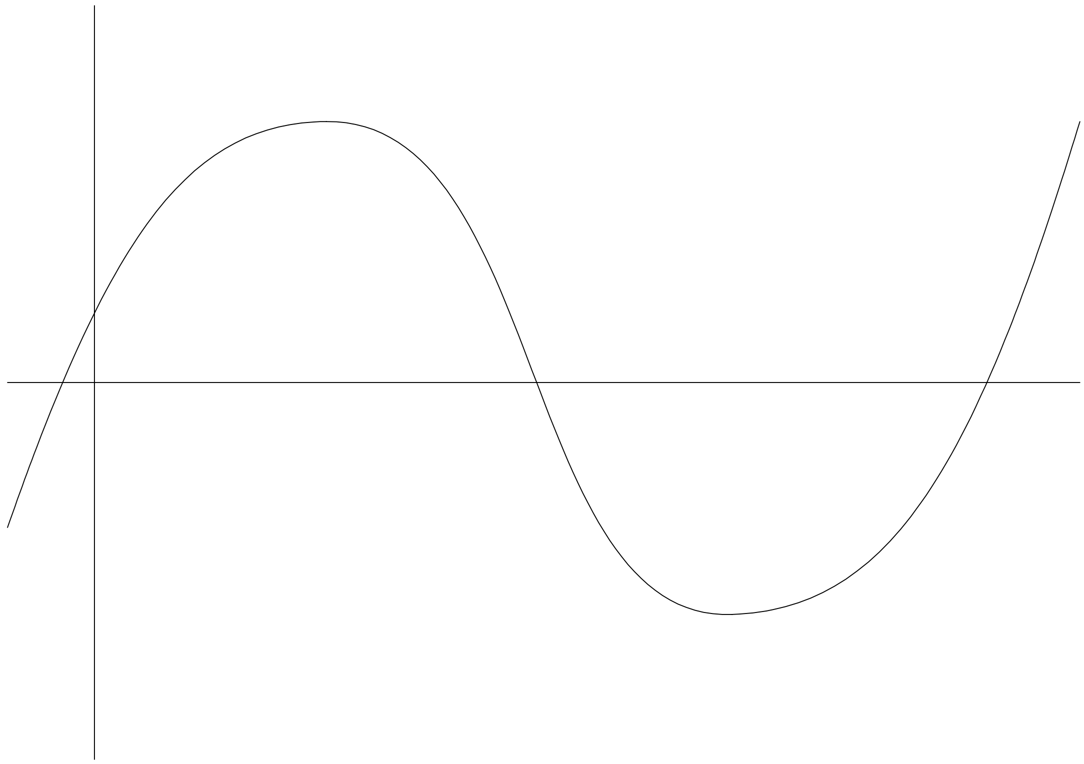
(9) (5 points) For the following function draw the graph of the derivative on the same axis.



(10) (10 points) Draw graphs of functions with the following properties

(a) $f'(x) > 0,\ f''(x) < 0$

(b) f is increasing at an decreasing rate.

(c) $f(1) = 2,\ f'(1) = 0,\ f''(x) < 0.$

(d)
- $f'(x) > 0$ for $1 < x < 4$
- $f'(x) < 0$ for $x < 1$ and for $4 < x$.

(11) (10 points) Use your calculator to sketch a graph of $y = x^3 + 3x^2 + x - 1$ and to find all the local maximizers and local minimizers of the function.

(a) Sketch of graph:

(b) Local maximizers: ______________________

(c) Local minimizers: ______________________

(12) (5 points) The energy expended by a bird per day, E, depends on the time spent forging for food per day, F hours. Foraging for a shorter time requires better territory, which then requires more energy for its defense. Find the foraging time that minimizes expenditure if

$$E = .5F + \frac{1.5}{F^2}.$$

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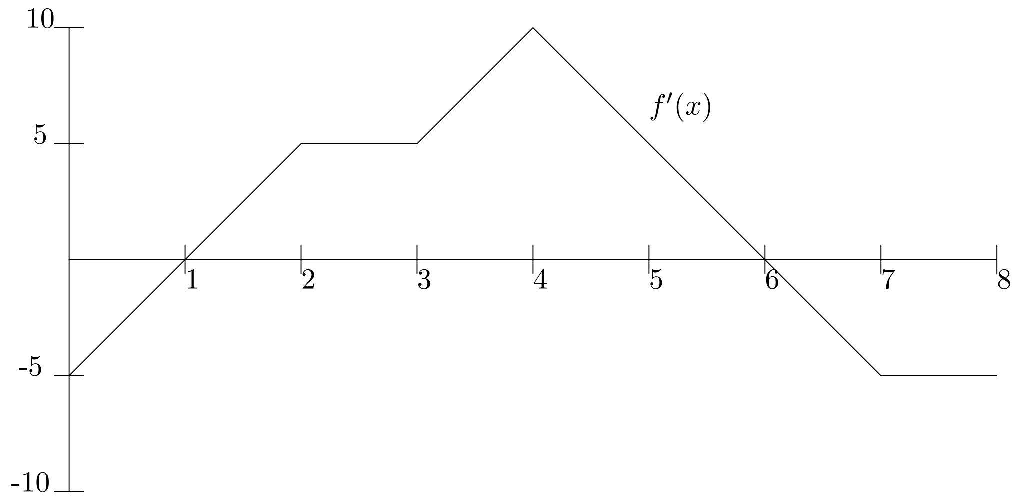
(13) (10 points)The following is a graph of the derivative $f'(x)$ of a function f.



(a) If $f(0) = 5$ complete the following table

x	1	2	3	3	5	6	7	8
$f(x)$								

(b) What is the maximum of $f(x)$? ____________________

(c) What is the maximizer of $f(x)$) ____________________

(14) (5 points) A water tank springs a leak. The rate R the water is coming out of the tank is given by the following table:

t (minutes after the leak starts)	0	5	10	15	20
R (gallons / minute)	32	28	25	23	19

Give upper, lower, and best guess estimates, of the total amount of oil that has leaked out in the first 15 minutes of the leak.

Upper ____________________

Lower ____________________

Best Guess ____________________

(15) (10 Points)
(a) Graph $y = x^2 + x$ and $y = x + 1$ on the same axis.

(b) At what points do the graphs of $y = x^2 + x$ and $y = x + 1$ intersect? ________________

(c) What is the area between $y = x^2 + x$ and $y = x + 1$ ________________

(16) (15 points) Find the following indefinite integrals (i.e. antiderivatives).
(a) $\int (5x^3 + 4x^2 + 2x - 1)\, dx =$

(b) $\int \left(\sqrt{t} + \frac{3}{t^5}\right)\, dt =$

(c) $\int e^x\, dx =$

(d) $\int e^{3t}\, dt =$

(e) $\int \left(e^\pi + \frac{1}{x}\right)\, dx =$

(17) (20 Points.) Find the derivatives of the following functions.
(a) $y = 5x^3 - 6x^2 + 3x - 4$

$y' =$

(b) $w = \dfrac{4}{z^5} + 5\sqrt{z}$

$w' =$

(c) $f(x) = -8e^x$

$f'(x) =$

(d) $Q = 5e_{-3P}$

$\dfrac{dQ}{dP} =$

(e) $y = 4\ln(x^2 + x)$

$y' =$

(f) $y = x^2 e^x$

$y' =$

(g) $w = 6(z^4 + z)^{20}$

$w' =$

(h) $y = \dfrac{e^x + 1}{e^x - 1}$

$y' =$

(18) (10 points) Compute the following

(a) $\displaystyle\int_{-1}^{2} \sqrt{e^{2x} + 3}\, dx =$

(b) The derivative of $f(x) = \dfrac{\ln(x) + 2}{x + 1}$ at the point where $x = 2.3$.

$f'(2.3) =$ ____________________

Have a nice holiday!