Mathematics 122 Test Final Name:______________________________

You are to use your own calculator, no sharing.

Show your work to get credit. This means that if you use your calculator to solve a problem, then you have to write a sentence telling how you used it to do the calculations. (That is if you graphed it and zoomed in then say that is what you did etc.

1. (25 points) Find the following derivatives:

(a) $y = 3x^4 - 7x^2 + x - 3$ $y' =$ ____________________

(b) $R(t) = 5e^t$ $R'(t) =$ ____________________

(c) $H(t) = \sqrt{t + 2t^3}$ $H'(t) =$ ____________________

(d) $P(r) = e^{3r^2+2r}$ $P'(r) =$ ____________________

(e) $I(x) = \dfrac{1}{x^2 + 7}$ $I'(x) =$ ____________________

2. (5 points) Find the equation of the line through $(2, -3)$ with slope $\dfrac{4}{3}$.

3. (10 points) Find the equation of the tangent line to $y = x^2 - x$ at the point where $x = 2$

4. (15 points) In 1985 there were 300 acres of kudzu growing in Richland county and by 1995 this had increased to 680 acres. Assume that the number of acres of kudzu is a linear function of time (measured in years)

   (a) What is the average rate of change in the number of acres of kudzu between 1985 and 1995? Give the units on your answer.

   ____

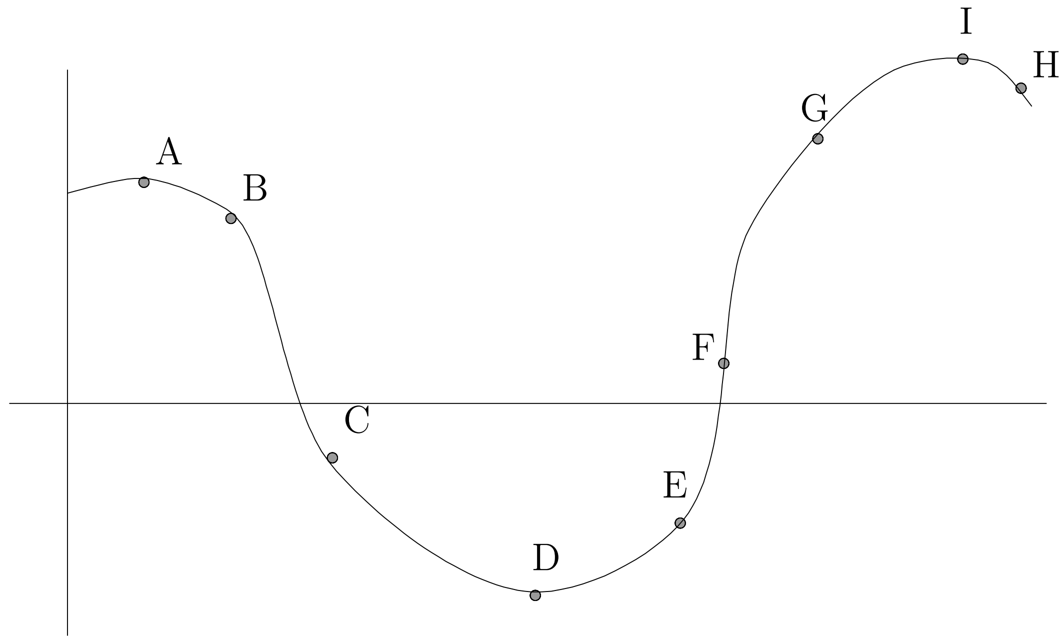
   (b) Predict how many acres of Kudzu will be growing in Richland county in the year 2000.

   ____

   (c) Make a prediction of what year there will be 1000 acres of kudzu in Richland county.

   ____

5. (10 Points) Let $f = f(x)$ have the following graph.



(a) At which of the labeled points is $f'(x) > 0$? ________________

(b) At which is the labeled points is $f'(x) < 0$? ________________

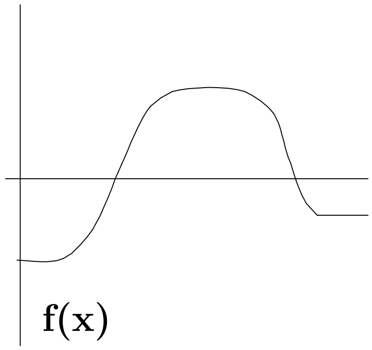
(c) At which is the labeled points is $f'(x) = 0$? ________________

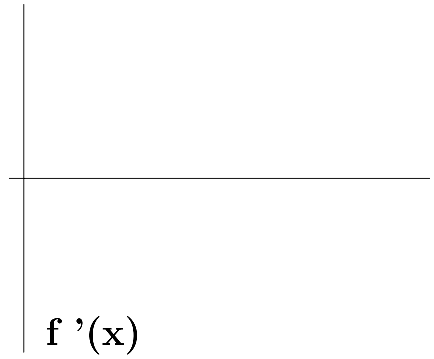
(d) At which of the labeled points if $f(x)$ largest? ________________

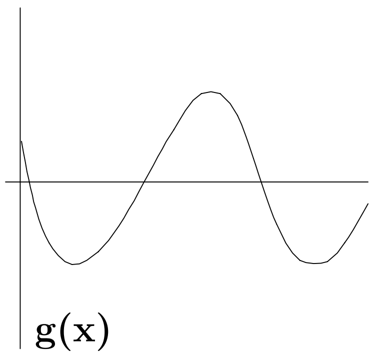
(e) At which of the labeled points is $f'(x)$ largest? ________________

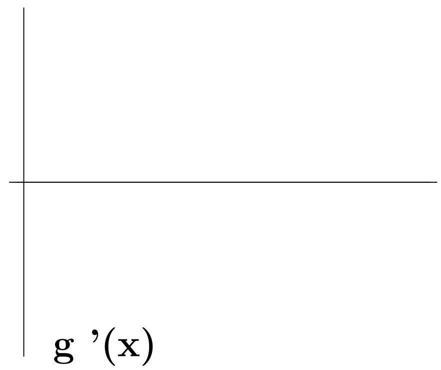
6. (5 points) Let $f(t) = (1.3)^t/(1+t^2)$. Then estimate $f'(1)$ $f'(1) \approx$ ________________

7. (10 Points) Below are the graphs of $y = f(x)$ and $y = g(x)$. Sketch the graphs of $y = f'(x)$ and $y = g'(x)$.









8. (10 Points) Draw the graph of a function $y = f(x)$ so that $f'(x) > 0$ for $0 < x < 2$, $f'(x) < 0$ for $2 < x < 3$ and $f'(x) > 0$ for $3 < x < 5$.

9. (15 Points) Let the function $u = f(t)$ have its values as in the following table:

t	.5	1.0	1.5	2.0	2.5
$f(t)$	1.25	2.0	2.25	2.0	1.25

(a) What is that average rate of change of f on the interval from $t = 1.0$ to $t = 2.5$?

____

(b) What is an estimate for $f'(2.5)$? ____________________

(c) What is a good estimate for $f(3)$? ____________________

10. (15 Points) Draw Graphs of functions that satisfy the following:

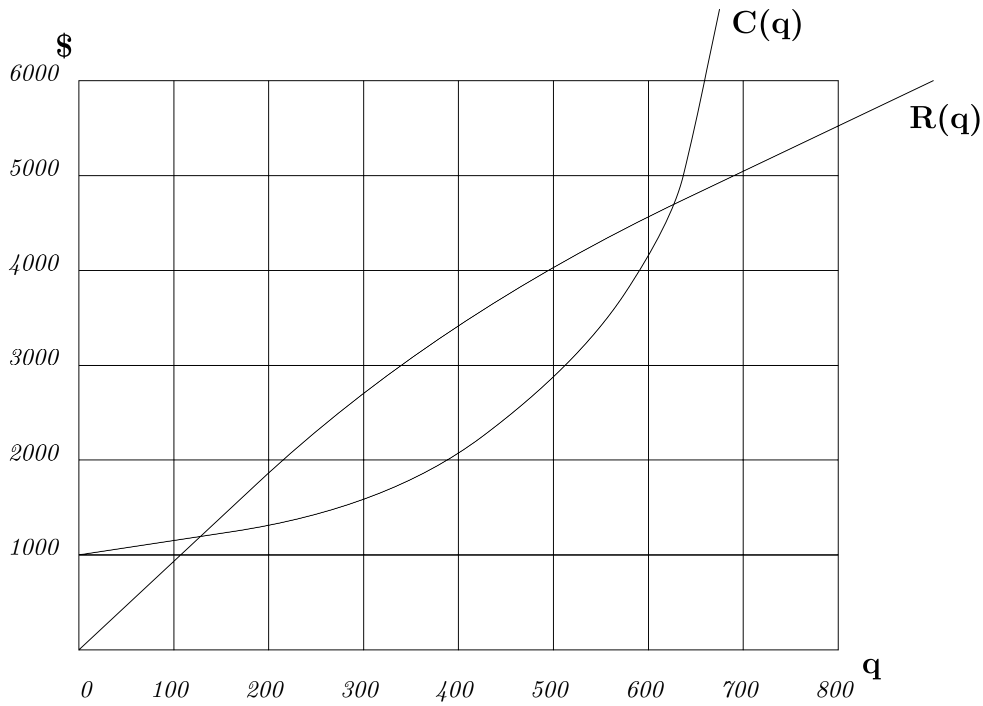
(a) Is increasing at an increasing rate.

(b) Is decreasing and concave up.

(c) Has $f'(x) > 0$ and $f''(x) < 0$

(d) has $f''(x) < 0$ and $f'(1) = 0$.

11. (15 Points) A group of students decide to market a guide to the bars of Five Points (which they expect to sell better than the manual to a calculus book they had previously sold). The following graph shows the cost, $C(q)$, and the revenue, $R(q)$, from producing a quantity q of the guides.



(a) About how much were the startup costs of the for producing the guides?

____

(b) From the graph roughly how much is the marginal revenue $R'(100)$ of producing 100 guides?

____

(c) If the students are producing 300 guides then is it in their interests to produce more guides? Write a sentence or two explaining your answer.

(d) If the students are producing 600 guides then is it in their interest to product more guides? Write a sentence or two explaining your answer.

(e) Make a guess the number of guides they should product to maximize their profit.

____

12. (15 points) Compute the following (you should use your calculator)

(a) $\displaystyle\int_1^3 17(1.4)^s\, ds$ ____________________

(b) $\displaystyle\int_{-2}^2 (x^4 - 2x)\, dx$ ____________________

(c) $\displaystyle\int_0^3 \frac{1}{4+x^2}\, dx$ ____________________

(d) $\displaystyle\int_1^4 \frac{1-2^t}{1+2^t}\, dt$ ____________________

13. (5 points) A function $f(t)$ has values given by the table:

t	.5	1.0	1.5	2.0	2.5
$f(t)$	1.25	2.0	2.25	2.5	2.65

Estimate $\displaystyle\int_{.5}^{2.5} f(t)\, dt$.

$\displaystyle\int_{.5}^{2.5} f(t)\, dt \approx$ ____________________

14. (15 points) Let $C(t)$ be the cost/day to cool a house, starting at $t = 0$ on June 1, 1995. Assume that $C(t)$ is given by

$$C(t) = \begin{cases} 1.5, & 0 \le t < 45 \\ 2.0, & 45 \le t \le 105 \\ 1.0 & 105 < t \le 120. \end{cases}$$

(a) What are the units on $\displaystyle\int_0^{120} C(t)\,dt$?

Units are: ____________________

(b) What is the interpretation of $\displaystyle\int_0^{120} C(t)\,dt$?

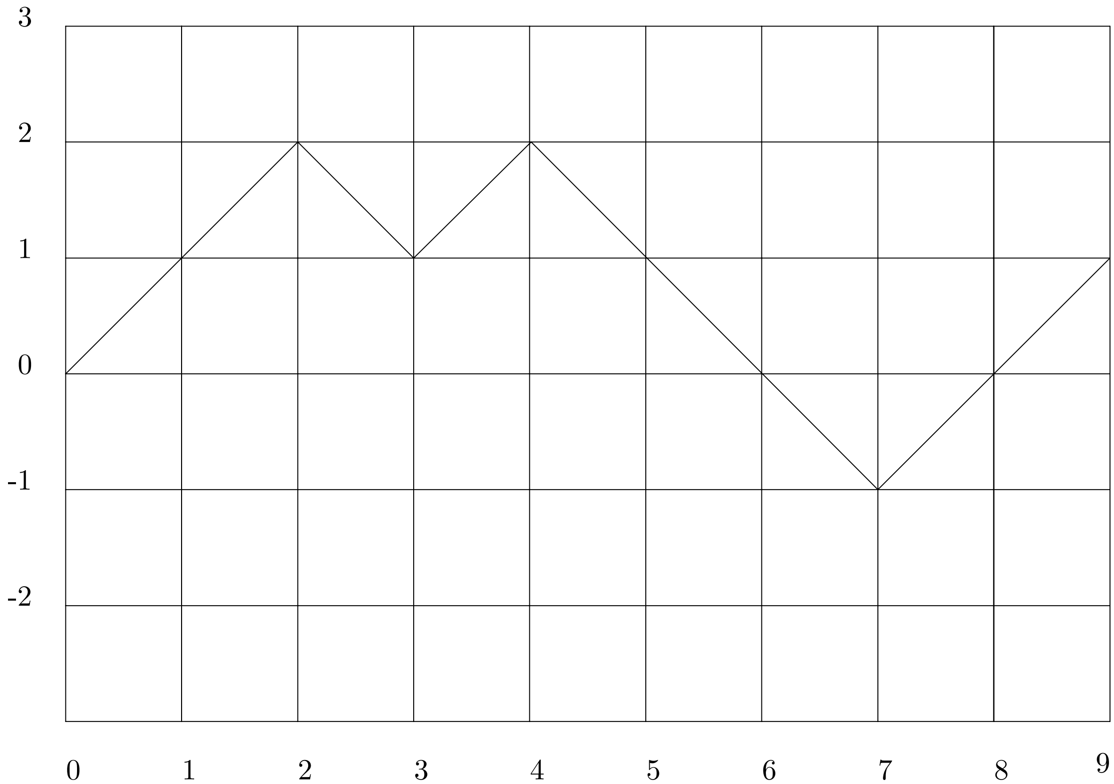
(c) What is the value of $\int_0^{120} C(t)\,dt$?

$\displaystyle\int_0^{120} C(t)\,dt =$ ____________________

(d) What is the average value of $C(t)$ on the interval $0 \le t \le 90$?

Average = ____________________

15. (20 points) The graph below is of the derivative of a function:



Graph of $y = F'(t)$

If we know that $F(0) = 2$ compute the following

(a) $\displaystyle\int_0^9 F'(t)\,dt$ $\qquad\displaystyle\int_0^9 F'(t)\,dt =$ __________

(b) $F(3)$ $\qquad F(3) =$ __________

(c) $F(9)$ $\qquad F(9) =$ __________

(d) The minimum value of $F(t)$. $\qquad$ Min.= __________

(e) The point where the minimum occurs. $\qquad$ Min. at $t =$ __________

16. (10 points) Let $f(x) = x^3 - 6x^2 + 9x$.

(a) Where is the local maximum of $f(x)$? ____________________

(b) Where is the local minimum of $f(x)$? ____________________