

Contents

1	Normed Spaces	1
1.1	Examples of Normed Spaces	1
1.2	Properties of Normed Spaces	1
1.3	Quotients and Sums of Normed Spaces	1
1.4	Problems	1
1.5	Remarks and Overviews	1
2	Functionals and Operators	3
2.1	Examples and Properties of Continuous Linear Operators	3
2.2	Dual spaces and their Representations	3
2.3	Compact Operators	3
2.4	Interpolation of Operators on L^p -spaces	3
2.5	Problems	3
2.6	Remarks and Overviews	3
3	The Hahn Banach Theorem and its Consequences	5
3.1	Extension of Functionals	5
3.2	Separation of Convex Sets	5
3.3	Weak Convergence and Reflexivity	5
3.4	Adjoint Operators	5
3.5	Differentiation of nonlinear mappings	5
3.6	Problems	5
3.7	Remarks and Overviews	5
4	The Fundamental Theorems for Operators on Banach Spaces	7
4.1	Preparation: Baire Category Theorem	7
4.2	Uniform Boundedness Principle	7
4.3	Open Mapping Theorem	7
4.4	Closed Graph Theorem	7
4.5	Closed Images Theorem	7
4.6	Projections onto Banach Spaces	7

4.7	Fixed Point Theorem	7
4.8	Problems	7
4.9	Remarks and Overviews	7
5	Hilbert Spaces	9
5.1	Defintions and Examples	9
5.2	Fourier Transform and Sobolev Spaces	9
5.3	Orthogonality	9
5.4	Orthonormal Bases	9
5.5	Operators on Hilbert Spaces	9
5.6	Problems	9
5.7	Remarks and Overviews	9
6	Spectral Theory for Compact Operators	11
6.1	Spectrum of Bounded Operators	11
6.2	Riesz's Theorem	11
6.3	Compact Operators on Hilbert Spaces	11
6.4	Application to Integral Equations	11
6.5	Nuclear Operators	11
6.6	Hilbert Schmidt Operators	11
6.7	Problems	11
6.8	Remarks and Overviews	11
7	Spectral Decompostion of Self-adjoint Operators	13
7.1	Spectral Theorem for Bounded Operators	13
7.2	Unbounded Operators	13
7.3	Spectral Theorem for Unbounded Operators	13
7.4	Operator semigroups	13
7.5	Problems	13
7.6	Remarks and Overviews	13
8	Locally Convex Spaces	15
8.1	Definition of Locally Convex Spaces; Examples	15
8.2	Continuous Functionals and the Hahn Banach Theorem	15
8.3	Weak Topology	15
8.4	Extreme points and the Krein Milman Theorem	15
8.5	Introduction to Distribution Theory	15
8.6	Problems	15
8.7	Remarks and Overviews	15
9	Banach Algebras	17
9.1	Basic Definitions and Examples	17
9.2	Gelfand Representation Theorem	17
9.3	C^* Algebras	17
9.4	Problems	17
9.5	Remarks and Overviews	17

A	Measure and Integration Theory	19
A.1	The Lebesgue Integral for Function on an Interval	19
A.2	The d -dimensional Lebesgue Measure and Abstract Integration	19
A.3	Convergence Theorems	19
A.4	Signed and Complex Measures	19
B	Metric and Topological Spaces	21
B.1	Metric Spaces	21
B.2	Topological Spaces	21

Chapter 1

Normed Spaces

1.1 Examples of Normed Spaces

1.2 Properties of Normed Spaces

1.3 Quotients and Sums of Normed Spaces

1.4 Problems

1.5 Remarks and Overviews

Chapter 2

Functionals and Operators

2.1 Examples and Properties of Continuous Linear Operators

2.2 Dual spaces and their Representations

2.3 Compact Operators

2.4 Interpolation of Operators on L^p -spaces

2.5 Problems

2.6 Remarks and Overviews

Chapter 3

The Hahn Banach Theorem and its Consequences

3.1 Extension of Functionals

3.2 Separation of Convex Sets

3.3 Weak Convergence and Reflexivity

3.4 Adjoint Operators

3.5 Differentiation of nonlinear mappings

3.6 Problems

3.7 Remarks and Overviews

Chapter 4

The Fundamental Theorems for Operators on Banach Spaces

4.1 Preparation: Baire Category Theorem

4.2 Uniform Boundedness Principle

4.3 Open Mapping Theorem

4.4 Closed Graph Theorem

4.5 Closed Images Theorem

4.6 Projections onto Banach Spaces

4.7 Fixed Point Theorem

4.8 Problems

4.9 Remarks and Overviews

Chapter 5

Hilbert Spaces

5.1 Defintions and Examples

5.2 Fourier Transform and Sobolev Spaces

5.3 Orthogonality

5.4 Orthonormal Bases

5.5 Operators on Hilbert Spaces

5.6 Problems

5.7 Remarks and Overviews

Chapter 6

Spectral Theory for Compact Operators

6.1 Spectrum of Bounded Operators

6.2 Riesz's Theorem

6.3 Compact Operators on Hilbert Spaces

6.4 Application to Integral Equations

6.5 Nuclear Operators

6.6 Hilbert Schmidt Operators

6.7 Problems

6.8 Remarks and Overviews

Chapter 7

Spectral Decomposition of Self-adjoint Operators

7.1 Spectral Theorem for Bounded Operators

7.2 Unbounded Operators

7.3 Spectral Theorem for Unbounded Operators

7.4 Operator semigroups

7.5 Problems

7.6 Remarks and Overviews

Chapter 8

Locally Convex Spaces

8.1 Definition of Locally Convex Spaces; Examples

8.2 Continuous Functionals and the Hahn Banach Theorem

8.3 Weak Topology

8.4 Extreme points and the Krein Milman Theorem

8.5 Introduction to Distribution Theory

8.6 Problems

8.7 Remarks and Overviews

Chapter 9

Banach Algebras

9.1 Basic Definitions and Examples

9.2 Gelfand Representation Theorem

9.3 C^* Algebras

9.4 Problems

9.5 Remarks and Overviews

Appendix A

Measure and Integration Theory

A.1 The Lebesgue Integral for Function on an Interval

A.2 The d -dimensional Lebesgue Measure and Abstract Integration

A.3 Convergence Theorems

A.4 Signed and Complex Measures

Appendix B

Metric and Topological Spaces

B.1 Metric Spaces

B.2 Topological Spaces

