MARK BOX		
PROBLEM	POINTS	
1 (1.1 – 1.15)	30	
2 (2.1 – 2.20)	20	
TOTAL	50	

Signature: ______________________________

Name (printed): ______________________

INSTRUCTIONS:

(1) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.

(2) This is a closed book/notes exam covering (from *Introduction to Real Analysis*, 2nd ed., by Stoll): § 2.5, 2.6, 2.7, 3.1, 3.2, 3.3, 4.1 .

NOTATION for throughout exam:

- $\{p_n\}_{n=1}^{\infty}$ and $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are sequences from $\mathbb{R}$.
- $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are infinite series in $\mathbb{R}$.
- A and B and E are subsets of $\mathbb{R}$.
- $\overline{E}$ denotes the closure of E.
  E' denotes the set of limit points of E.
  $Int(E)$ denotes the set of interior points of E.
- *iff* is short for *if and only if*

1. Fill in the blanks. When it says, *by definition*, be sure to give the definition and not some equivalent formulation.

1.1 By definition, $\varliminf_{n\to\infty} p_n = \sup_{k\in\mathbb{N}} \inf \{$ ______________________________ $\}$.

1.2 By definition, $\{p_n\}_n$ is a *Cauchy sequence* provided

____ .

1.3 The sequence $\{s_n\}_{n=1}^{\infty}$ of *partial sums* of a series $\sum_{n=1}^{\infty} a_n$ is defined by $s_n =$ ____________ .

1.4 By definition (and using the notation from the above problem), the series $\sum_{n=1}^{\infty} a_n$ converges iff ____ .

1.5 Some big theorem in the book says that $\sum_{n=1}^{\infty} a_n$ converges if and only if it satisfies the *Cauchy Criterion*. Write out precisely what it means for $\sum_{n=1}^{\infty} a_n$ to satisfy the Cauchy Criterion (hint: this is an ϵ-N criterion).

____ .

1.6 By definition, p is an *interior point* of E if and only if $p \in E$ and

____ .

1.7 By definition, E is *open* iff ____.

1.8 By definition, E is *closed* iff ____.

1.9 By definition, $p \in \mathbb{R}$ is a *limit point* of E iff ________ $\epsilon > 0$ ______________________________.

1.10 $p \in \mathbb{R}$ is a *isolated point* of E iff ________ $\epsilon > 0$ ______________________________ .

1.11 By definition, a subset D of $\mathbb{R}$ is *dense* in $\mathbb{R}$ iff ______________________________ .

1.12 If $E = (0, 1] \cup \{2\}$ then: $IntE =$ __________ and $E' =$ __________ and $\overline{E} =$ __________ and Isolated points of $E =$ __________

1.13 If $E = \mathbb{Q}$ then: $IntE =$ __________ and $E' =$ __________ and $\overline{E} =$ __________ and Isolated points of $E =$ __________

1.14 By definition, E is compact iff for each collection $\{O_\alpha\}_{\alpha\in\Gamma}$ of open sets such that $E \subset \cup_{\alpha\in\Gamma} O_\alpha$ there exists a finite collection $\{\alpha_i\}_{i=1}^N$ from Γ such that ____________________ .

1.15 Let $f : E \to \mathbb{R}$ and $p \in E'$ and $L \in \mathbb{R}$. By definition, $\lim_{x\to p} f(x) = L$ iff

____ .

2. If the statement is true, then circle T. If the statement is false, then circle F.

2.1 **T F** $\overline{\lim_{n\to\infty}} \mid p_n \mid = 0$ if and only if $\lim_{n\to\infty} p_n = 0$

2.2 **T F** $\underline{\lim_{n\to\infty}} \ (-a_n \ ) \ = \ - \overline{\lim_{n\to\infty}} \ a_n$ (here, $\{a_n\}$ is bounded)

2.3 **T F** $\overline{\lim_{n\to\infty}} (a_n + b_n) \leq \overline{\lim_{n\to\infty}} \ a_n + \overline{\lim_{n\to\infty}} \ b_n$ (here, $\{a_n\}$ and $\{b_n\}$ are bounded)

2.4 **T F** $\underline{\lim_{n\to\infty}} \ a_n + \underline{\lim_{n\to\infty}} \ b_n \leq \underline{\lim_{n\to\infty}} (a_n + b_n)$ (here, $\{a_n\}$ and $\{b_n\}$ are bounded)

2.5 **T F** $\{p_n\}$ is Cauchy if and only if $\{p_n\}$ converges to some (finite) real number.

2.6 **T F** Each contractive sequence in $\mathbb{R}$ converges in $\mathbb{R}$.

2.7 **T F** If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

2.8 **T F** Let $a_n \geq 0$ for each $n \in \mathbb{N}$. Then $\sum_{n=1}^{\infty} a_n$ converges if and only if its sequence $\{s_n\}_{n=1}^{\infty}$ of partial sums is bounded above.

2.9 **T F** If each A_n is open, then $\cup_{n=1}^{\infty} A_n$ is open.

2.10 **T F** If each A_n is closed, then $\cap_{n=1}^{\infty} A_n$ is closed.

2.11 **T F** $\mathbb{Q}$ is dense in $\mathbb{R}$.

2.12 **T F** $\overline{E} = E \cup E'$.

2.13 **T F** E is closed iff $E = \overline{E}$.

2.14 **T F** $\overline{(A \cup B)} = \overline{A} \ \cup \ \overline{B}$

2.15 **T** **F** E is open iff $E = Int(E)$.

2.16 **T** **F** $Int(A \cap B) = Int(A) \ \cap \ Int(B)$

2.17 **T** **F** If E is an open subset of $\mathbb{R}$ then there exits a finite or countable collection $\{I_n\}$ of pairwise disjoint open intervals such that $E = \cup_n \ I_n$.

2.18 **T** **F** E is compact if and only if E is closed and bounded.

2.19 **T** **F** Let K be a nonempty compact subset of $\mathbb{R}$. Then each sequence in K has a subsequence that converges to a point in K.

2.20 **T** **F** The Cantor set is open.