MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
Total	20	
%	100	

Math 554/703i.002 Prof. Girardi

Fall 98 Exam 32 11/24/98

NAME: ____________________________

INSTRUCTIONS:

1. Write a NEAT FORMAL proof to **2** of the **4** problems.
   I am doing problem numbers: ____ .
2. Use your own paper:
   a. write on only one side of the page
   b. begin each problem on a new page
   c. put your name on each page.
2. The MARK BOX indicates the problems along with their points.
   Check that your copy of the exam has all of the problems.
3. During this test, do not leave your seat.
   If you have a question, raise your hand.
4. This closed book/notes exam covers (*Intro. to Real Analysis*, 1st ed., by Stoll): Sections 2.4 – 2.6.

Problem Source:

1. *Fundamental Ideas of Analysis* by Michael Reed: § 2.6 # 3 & 4.
2. look at problem § 2.5 # 7
3. I made it up today.
4. Standard question, we mentioned it in class.

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1. Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be bounded sequences in $\mathbb{R}$ and $a, c, d \in \mathbb{R}$.

1a. By definition, $\{a_n\}$ CONVERGES to a if

____ .

By definition, d is a SUBSEQUENTIAL LIMIT POINT of $\{b_n\}$ if

____ .

1b. Let $\{a_n\}$ converge to a and d be a subsequential limit point of $\{b_n\}$. Show that $a \cdot d$ is a subsequential limit point of $\{a_n \cdot b_n\}$.

1c. Let c be a subsequential limit point of $\{a_n\}$ and d be a subsequential limit point of $\{b_n\}$. Is $c + d$ necessarily a subsequential limit point of $\{a_n + b_n\}$? Prove so or give a specific counterexample.

2. Let $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ be BOUNDED sequences of POSITIVE real numbers. Show that

$$\overline{\lim_{n\to\infty}} (a_n b_n) \leqslant \left(\overline{\lim_{n\to\infty}}\, a_n\right)\left(\overline{\lim_{n\to\infty}}\, b_n\right) .$$

Need equality hold in the above inequality? Prove so or give a specific counterexample. (Recall, the sequences are bounded and positive-termed).

3. Let $\{x_n\}$ and $\{y_n\}$ be sequences in $\mathbb{R}$. Let $\{\varepsilon_n\}$ be a sequence of POSITIVE real numbers satisfying

$$\lim_{n\to\infty} \varepsilon_n = 0 .$$

3a. By definition, $\{x_n\}$ is CAUCHY if

____ .

3b. A BIG-TIME theorem says that $\{x_n\}$ is Cauchy if and only if

____ .

3c. Let $\{x_n\}$ be Cauchy and furthermore, for each $n \in \mathbb{N}$,

$$|x_n - y_n| < \varepsilon_n .$$

Show that $\{y_n\}$ is Cauchy. You may use **3a** but you may NOT use **3b**.

4. Let $\{x_n\}$ be a bounded sequence in $\mathbb{R}$.

4a. By definition, $\overline{\lim}_{n\to\infty} x_n = \lim_{k\to\infty} b_k$ where $b_k = \sup\{$______________________$\}$.

4b. Clearly show that there exists a subsequence $\{x_{n_k}\}_{k=1}^{\infty}$ of $\{x_n\}_{n=1}^{\infty}$ satisfying

$$\lim_{k\to\infty} x_{n_k} = \overline{\lim_{n\to\infty}}\, x_n .$$

HINT: Use the definition from **4a** to construct such a subsequence.