

MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
Total	40	
%	100	

Math 554/703i.002 Prof. Girardi

Fall 98 Exam 1 9/24/98

NAME: _____

INSTRUCTIONS:

1. Write a NEAT FORMAL proof to **4** of the **5** problems.
I am doing problem numbers: _____ .
2. Use your own paper:
 - a. write on only one side of the page
 - b. begin each problem on a new page
 - c. put your name on each page.
2. The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
3. During this test, do not leave your seat.
If you have a question, raise your hand.
4. This closed book/notes exam covers (*Intro. to Real Analysis*, 1st ed., by Stoll):
Sections 1.1 – 1.6.

Problem Source:

1. hand in problem § 1.1 # 11
2. 1996 exam # 1 problem # 3
3. an example from class: § 1.4 # 8
4. in class problem § 1.4 # 11
5. 1996 exam # 1 problem # 5

1. Suppose A and C are subsets of X while B and D are subsets of Y . Prove that

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D) .$$

2. Consider a function $f: \{0, 1, 2, \dots\} \rightarrow \mathbb{R}$ that is defined recursively by $f(0) = 5$ and $f(1) = 1$ and

$$f(n+1) = f(n) + 2f(n-1)$$

and for $n \geq 1$. Use math induction to show that

$$f(n) = 2^{n+1} + 3(-1)^n$$

for all integers $n \geq 0$.

3. Let E be a non-empty subset of \mathbb{R} that is bounded above and set

$$U = \{\beta \in \mathbb{R}: \beta \text{ is an upper bound of } E\} .$$

Let

$$\gamma_1 = \sup E$$

and

$$\gamma_2 = \inf U .$$

Show that $\gamma_1 = \gamma_2$.

4. Let A and B be non-empty subsets of \mathbb{R} with $A \subset B$. Prove that

$$\inf B \leq \inf A \leq \sup A \leq \sup B .$$

5. Let A and B be arbitrary sets with $A_1 \subset A$ and $B_1 \subset B$.

Consider a function $f: A \rightarrow B$.

- 5a. f is one-to-one provided that: if $a_1, a_2 \in A$ and $a_1 \neq a_2$, then _____ .
- 5b. The definition of $f^{-1}(B_1)$ says that: $a \in f^{-1}(B_1)$ if and only if _____ .
- 5c. Show that $A_1 \subset f^{-1}f(A_1)$.
- 5d. Show that f is one-to-one if and only if $D = f^{-1}f(D)$ for all subsets D of A .