

MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
TOTAL	40	
%	100	

NAME: _____

SSN: _____

INSTRUCTIONS:

- (1) For the *proof problems*, write a NEAT FORMAL proof. You may, if you want, include your *thinking scratch work* but you must also include your formal proof for full credit.
- (2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
- (3) This is a closed book/notes exam covering (from *Introduction to Real Analysis*, 3rd ed., by Bartle and Sherbert): Ch. 1, § 2.1–2.4 .

Problem Inspiration:

- (2) Example 1.2.4a (3) Example 2.4.1a (4) Hmwk 1.1.17a.

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1. If the statement is true, then circle T. If the statement is false, then circle F.
 A , B , and C are sets.
1. T F : $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$
 2. T F : $A \setminus (B \cap C) = (A \setminus B) \cap (A \setminus C)$
 3. T F : $f: A \rightarrow B$ is injective provided if $a_i \in A$ and $f(a_1) \neq f(a_2)$ then $a_1 \neq a_2$.
 4. T F : B is countably infinite if and only if there is a bijection from B onto \mathbb{N} .
 5. T F : B is countably infinite if and only if there is a bijection from B onto \mathbb{Q}
 6. T F : $\mathbb{N} \times \mathbb{N}$ is countable.
 7. T F : If $A \subseteq B$ and B is an infinite set, then A is an infinite set.
 8. T F : If $a, b \in \mathbb{R}$ and $a \neq b$, then there are ε -neighborhoods U of a and V of b so that $U \cap V = \emptyset$.
 9. T F : $\sup(A \cup B) = \sup A + \sup B$
 0. T F : Each nonempty set of real numbers has a supremum in \mathbb{R} .

2. For $n \in \mathbb{N}$, let $S_n := \sum_{k=1}^n k$, i.e.,

$$S_n := 1 + 2 + \dots + n .$$

Use math induction to prove that

$$S_n = \frac{n(n+1)}{2} \tag{2}$$

for each $n \in \mathbb{N}$.

- 3.** Let T be a nonempty subset of \mathbb{R} that is bounded above and $a \in \mathbb{R}$. Define the set $a + T$ by

$$a + T := \{a + t \in \mathbb{R} : t \in T\} .$$

Prove that

$$\text{lub}(a + T) = a + \text{lub} T . \tag{3}$$

Some helpful notation to get you started ... let $u := \text{lub} T$.

4. Consider a function $f: X \rightarrow Y$. Let $A \subseteq X$ and $B \subseteq Y$.

Let C and D be arbitrary (i.e., just any old) sets.

4a. By definition of (direct) image, $y \in f(A) \Leftrightarrow$ _____ .

4b. By definition of inverse (pre-) image, $x \in f^{-1}(B) \Leftrightarrow$ _____ .

4c. By definition of relative complement, $z \in D \setminus C \Leftrightarrow$ _____ .

4d. Prove that $A \subseteq f^{-1}(f(A))$.

4e. Prove that $A = f^{-1}(f(A))$ if and only if $f(X \setminus A) \subseteq Y \setminus f(A)$.