MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	
%	100	

Math 550A Prof. Girardi

Spring 99 Exam 2

3/20/99

NAME: ____________________________

INSTRUCTIONS:

1. To receive credit you must:
   a. WORK IN A LOGICAL FASHION, SHOW ALL YOUR WORK, and INDICATE YOUR REASONING
   b. begin each (numbered) problem on a new page and put your pages in the proper order
   c. when applicable, please box your answer.
2. The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
3. As indicated on the syballus:
   a. allowed is a calculator (but NOT a computer)
   b. allowed are the class handouts: table of integrals, calculus formula sheet, and informal summary (along with your personal scribbles on them)
   c. not allowed are books and other notes.
4. In theory, this is a 90 minute exam; however, you may take as much time as you want.
5. Compare your printed version of the exam to what appears on your computer screen. Last time several students' printed version did not print off some symbols (as minus and equal signs), which made the problem harder than it really was.
6. This exam covers (from *Vector Calculus* by Marsden & Tromba, 4^{th}ed.) : § 4.3 , § 4.4 , Ch 5 , Ch 6 .

This take-home exam is due THURSDAY 25 MARCH BY 10 AM.

According to the USC Student Handbook code of student academic responsibility, the first law of academic life is intellectual honesty.

I worked this take-home exam by *myself*, in accordance with the above instructions.

SIGNATURE: ____ .

1. The Problem 1 Link contains five vector fields ($vf_1 - vf_5$), five vector field sketches (a - e), and five flow line sketches (f - j). Using the below box, match each of the vector fields with its corresponding vector field sketch and flow lines.

vector field	vf_1	vf_2	vf_3	vf_4	vf_5
vector field sketch (a - e)					
flow lines (f - j)					

2. Recall that a vector field

$$\vec{G}(x,y,z) \;=\; \langle \;\; g_1(x,y,z) \;\;,\;\; g_2(x,y,z) \;\;,\;\; g_3(x,y,z) \;\rangle \;,$$

is irrotational if $\overrightarrow{\text{curl}}\,\vec{G} = \vec{0}$ and that a vector field

$$\vec{H}(x,y,z) \;=\; \langle \;\; h_1(x,y,z) \;\;,\;\; h_2(x,y,z) \;\;,\;\; h_3(x,y,z) \;\rangle \;,$$

is incompressible if div $\vec{H} = 0$. Take a moment to think to yourself (no need to tell me what you come up with) of *easy* (sufficient but not necessary) restrictions/conditions that you can put on the g_i's to make $\vec{G}$ irrotational and on the h_i's to make $\vec{H}$ incompressible. Now, consider the vector field

$$\vec{F}(x,y,z) \;=\; \langle \;\; x+y+z \;\;,\;\; y^2+1 \;\;,\;\; \ln(xyz) \;\rangle \;.$$

2a. Is $\vec{F}$ irrotational? Yes/No

2b. Is $\vec{F}$ incompressible? Yes/No

2c. Decompose $\vec{F}$ as the sum of an irrational vector field $\vec{G}$ and an incompressible vector field $\vec{H}$. I.e., express $\vec{F}$ as:

$$\vec{F}(x,y,z) \;=\; \vec{G}(x,y,z) \;+\; \vec{H}(x,y,z)$$

where $\vec{G}$ is irrational and $\vec{H}$ is incompressible.

3. Let **R** be the region in R^3 that is bounded by:

(1) $y = x^2 + 2$
(2) $3y - 4z = 0$
(3) $y = 4$
(4) $z = 0$
(5) $x = 0$.

Let

$$\mathbf{I} \;=\; \iiint_{\mathbf{R}} f(x,y,z)\,dV \;,$$

where $w = f(x,y,z)$ is a continuous function. Make a (rough) sketch of **R** and then express **I** in 6 different ways by taking into account each possible order of integration. Remark: to ease in grading, please do in the following order:
(a) dz dy dx (b) dz dx dy (c) dy dz dx (d) dy dx dz (e) dx dz dy (f) dx dy dz .
Hint: you will need 2 integrals for (c) and (f), 3 integrals for (d), and 1 integral for the others.

4. Let **P** be the parallelogram with vertices:

$$p_1 = (1,2) \quad , \quad p_2 = (3,1) \quad , \quad p_3 = (4,2) \quad , \quad p_4 = (2,3) \; .$$

Let **S** be the unit square, thus it is a paralelogram with vertices:

$$s_1 = (0,0) \quad , \quad s_2 = (1,0) \quad , \quad s_3 = (1,1) \quad , \quad s_4 = (0,1) \; .$$

4a. Find a one-to-one onto linear transformation $T\colon \mathbf{S} \to \mathbf{P}$ that satisfies $T(s_i) = (p_i)$ for each $i = 1, 2, 3, 4$.

4b. What is the Jacobian matrix J_T of T ?

4c. What is the determinate of J_T ?

4d. Given your answer to part 4c and the fact that S has area 1, what is the area of P?

5. Let $0 < a < b$ and $m > 0$. Let D be the solid region in $\mathbb{R}^3$ that is bounded by:

(1) $z = a$
(2) $z = b$
(3) $mz = x^2 \; + \; y^2$.

Let V be the volume of D.

5a. Make a rough sketch of D .

5b. Express V as a triple integral using rectangular coordinates (in the order of $dz\,dy\,dx$). Yuk ... don't want to integrate that so ....

5c. Find a transformation T from some region D^* onto D. Sketch your D^*. Briefly explain geometrically how T is transforming D^* onto D. Is your T one-to-one on D^*? Is your T one-to-one on the interior of D^*?

5d. Express V as a triple integral over D^* .

5e. Find V.