MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	
%	100	

Math 550A Prof. Girardi

Spring 99 Exam 1

2/13/98

NAME: ____________________________

INSTRUCTIONS:

1. To receive credit you must:
   a. WORK IN A LOGICAL FASHION, SHOW ALL YOUR WORK, and INDICATE YOUR REASONING
   b. begin each (numbered) problem on a new page and put your pages in the proper order
   c. when applicable, please box your answer.
2. The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
3. As indicated on the syballus:
   a. allowed is a calculator (but NOT a computer)
   b. allowed are the class handouts: table of integrals, calculus formula sheet, and informal summary (along with your personal scribbles on them)
   c. not allowed are books and other notes.
4. In theory, this is a 90 minute exam; however, you may take as much time as you want.
5. This exam covers (from *Vector Calculus* by Marsden & Tromba, 4^{th}ed.) : Chs. 1, 2, § 3.1, 4.1, 4.2 .

This take-home exam is due THURSDAY 18 FEBRUARY BY 10 AM.

According to the USC Student Handbook code of student academic responsibility, the first law of academic life is intellectual honesty.

I worked this take-home exam by *myself*. SIGNATURE: __________________ .

1. Find a parameterization of the line that passes through the point $(0,1,2)$ and is parallel to the intersection of the two planes: $x-4y+5z=17$ and $2x-3y+z=19$. Your solution should be of the form: $\vec{R}(t) = \langle\ x(t), y(t), z(t)\ \rangle$ where $\_ < t < \_$.

2. Let:

$$\vec{A} = \langle\ 1,1\ \rangle$$
$$\vec{B} = \langle\ 1,-1\ \rangle$$
$$\vec{C} = \langle\ 1,2\ \rangle$$

2a. Show that $\vec{A}$ and $\vec{B}$ are perpendicular.

2b. Draw a rough sketch illustrating that there are constants α and β such that $\vec{C} = \alpha\vec{A} + \beta\vec{B}$.

2c. Express $\vec{C}$ as a linear combination of $\vec{A}$ and $\vec{B}$, i.e., specifically find constants α and β so that $\vec{C} = \alpha\vec{A} + \beta\vec{B}$. Hint: Informal Summary 1.5.

3. Find an equation of the tangent plane to the graph of the function

$$f(x,y) = 10 - x^2 - y^2$$

at the point $(1,2,f(1,2))$. Your solution should be of the form: $ax+by+cz=d$.

4. A puffetta is moving along a curve that is parameterized by the path:

$$\vec{r}\,(t) \ = \ \langle\ \cos t + t\sin t\ ,\ -\sin t + t\cos t\ ,\ \frac{t^2}{2}\ \rangle$$

where $t \geqslant 0$.

4a. Find her velocity vector, acceleration vector, speed function, unit tangent vector $\vec{T}(t)$ and unit principle normal vector $\vec{N}(t)$.

4b. Express her acceleration as $\vec{a}(t) \ = \ a_T(t)\ \vec{T}(t) \ + \ a_N(t)\ \vec{N}(t)$.

5. Find the length of the curve $\vec{c}\,(t) = \langle\ \cos^3 t\ ,\ \sin^3 t\ \rangle$ where $0 \leqslant t \leqslant 2\pi$.

6. The temperature produced by a source located at the origin is given by:

$$T(x,y,z) \ = \ \exp\left[-\left(x^2+y^2+z^2\right)\right] .$$

6a. *Specifically* describe and sketch a typical level surface $T(x,y,z) \ = \ C$ of T. For which values of C's is the level surface non-empty? (Remark: level surfaces of temperature fields are called *isothermal surfaces*).

6b. At what point is it the warmest?

6c. What is the direction of the most rapid *decrease* in temperature at the point $(\ 1\ ,\ 2\ ,\ -4\ )$? Your solution should be of the form: In the direction of $\vec{u}$. And $\vec{u}$ should be a UNIT vector.

6d. Describe the directions(s) at the point $(\ 1\ ,\ 2\ ,\ -4\ )$ in which the rate of change of the temperature is no larger the $0.4\exp(-21)$.