MARK BOX		
PROBLEM	POINTS	
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

Math 550A Prof. Girardi

Spring 98 Exam 2

3/28/98

NAME: _____

INSTRUCTIONS:

1. To receive credit you must:
   a. WORK IN A LOGICAL FASHION, SHOW ALL YOUR WORK, INDICATE YOUR REASONING
   b. when applicable put your answer on/in the line/box provided
   c. if no such line/box is provided, then box your answer.
2. The MARK BOX indicates the problems along with their points.
   Check that your copy of the exam has all of the problems.
3. As indicated on the syballus:
   a. allowed is a calculator
   b. allowed are the class handouts: table of integrals, calculus formula sheet, and informal summary (along with your personal scribbles on them)
   c. not allowed are other notes and books.
4. This exam covers (from *Vector Calculus* by Marsden & Tromba, 4^{th}ed.) :
   § 4.3, § 4.4, Chs. 5 & 6 .

The sources of the questions on this exam are:

(1) taken from quiz 5 : § 4.4 problem 26, like problem 1 from 97 exam 2,
(2) problem 2 from 97 exam 2 (see textbook pgs 286 and 279),
(3) an example from class lectures (but with an easier integrand) that is like problem 4 from 97 exam 2,
(4) example 5 from page 344 of the textbook, like problem 3 from 97 exam 2,
(5) homework problem 5 from Ch. 6 Review, like problem 5 from 97 exam 2.

___ I am using a fancy calculator so often my "work" is not shown on items such as computing cross poducts.

___ I am not using a fancy calculator so my work is shown.

1. Consider the vector field

$$\vec{F}(x, y, z) = \langle\, x^2 + y^2 \;\;,\;\; -2xy,\;\;,\;\; 0\,\rangle \;\;.$$

1a. div $\vec{F}$ = _____ .

1b. $\overrightarrow{\text{curl}}\,\vec{F}$ = _____ .

1c. Is $\vec{F} \in C^1$? _____ (yes/no)
Explain your answer in (a few) complete sentences.

1d. Can $\vec{F}$ be a gradient vector field? _____ (yes/no)
Explain your answer in (a few) complete sentences.

2a. A figure on the next page shows some flow lines and moving regions for a fluid moving in the plane field velocity field $\vec{V}$.

Where is div $\vec{V} > 0$? _____ .

Where is div $\vec{V} < 0$? _____ .

Intuitively explain your answers in (a few) complete sentences.

2b. A figure on the next page shows the movement of a small rigid paddle wheel that is placed in moving fluid. The fluid has velocity field $\vec{V}$.

What can you say about the $\overrightarrow{\text{curl}}\,\vec{V}$? _____ .

Intuitively explain your answer in (a few) complete sentences.

Figure for Problem **2a.**

figure 4.4.9 page 286

Figure for Problem **2b.**

figure 4.4.7 page 279

3. Let D be the region in the xy-plane enclosed by the parallelogram with vertices: $(1,2)$, $(2,3)$, $(3,1)$, $(4,2)$.

3a. A linear transformation T that takes the unit square

$$D^* = \{(u,v)\colon 0 \leqslant u \leqslant 1, 0 \leqslant v \leqslant 1\}$$

in the uv-plane onto D is:

$T(u,v) \;=\; (\;\underline{\qquad\qquad\qquad\qquad}\;,\;\underline{\qquad\qquad\qquad\qquad}\;)\,.$

3b. Express $\iint_D (x+y)\,dydx$ as an integral over D^* (do not integrate).

$\iint_D (x+y)\,dydx \;=\; \int \qquad \int \qquad\qquad\qquad\qquad\qquad\qquad dudv \;\;.$

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4. Let D be the region in $\mathbb{R}^3$ that is bounded by

(1) $x = 0$
(2) $y = 0$
(3) $z = 2$
(4) $z = x^2 + y^2$

and is lying in the **first quadrant**. Make a (rough) sketch of D. Express the volume V of D as two triple integrals, with the order of integration as indicated below. You do NOT need to actually perform the integration.

4a. $V = \int \quad \int \quad \int \quad 1 \; dz \; dy \; dx \;$.

4b. $V = \int \quad \int \quad \int \quad 1 \; dx \; dy \; dz \;$.

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5. Let D be the region in $\mathbb{R}^3$ that is inside the surfaces $x^2+y^2 = z$ and $x^2+y^2+z^2 = 2$. Let T be the transformation:

$$T(r, \theta, z) = (r\cos\theta, r\sin\theta, z) \ .$$

5a. Then $T(D^*) = D$ and T is 1-to-1 on the interior of the **z-simple** region D^* where:

$$D^* \ = \ \{(r, \theta, z)\colon \ \_\_\_\_\_ \leqslant z \leqslant \_\_\_\_\_ \ , \ \_\_\_\_\_ \leqslant r \leqslant \_\_\_\_\_ \ , \ \_\_\_\_\_ \leqslant \theta \leqslant \_\_\_\_\_ \ \} \ .$$

5b. Is T 1-to-1 on D^*? _____ (yes/no)
Explain your answer in (a few) complete sentences with the aid of rough sketch of D and D^*.

5c. Express the volume V of D as a triple integral (you do not need to actually perform the integration):

$$V = \int \qquad \int \qquad \int \qquad dz \ dr \ d\theta \ \ .$$