MARK BOX		
PROBLEM	POINTS	
1	18	
2 a–f	18	
3	16	
4	16	
5	16	
6	16	
Total	100	

Math 550A Prof. Girardi

Spring 97 Final Exam

5/3/97

NAME: ____________________

INSTRUCTIONS:

1. To receive credit you must:
   a. WORK IN A LOGICAL FASHION, SHOW ALL YOUR WORK, INDICATE YOUR REASONING
   b. when applicable put your answer on/in the line/box provided
   c. if no such line/box is provided, then box your answer.
2. The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
3. As indicated on the syballus:
   a. allowed is a calculator
   b. allowed are the class handouts: table of integrals, calculus formula sheet, and informal summary (along with your personal scribbles on them)
   c. not allowed are other notes and books.
4. This exam covers (from *Vector Calculus* by Marsden & Tromba, 4^{th}ed.) : the whole thing .

WARNING:

Problems are NOT ordered by increasing level of difficulty.
Problems are ordered by the ordering in which we covered the topic in class.

The sources of the questions on this exam are:

(1) typical of § 1.3
(2) just me thinking
(3) from your homework
(4) from Edwards&Penny, 3^{rd} ed., pg 885
(5) from your homework
(6) like problems from previous exams

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1. Planes

1a. An equation, in the form of $ax + by + cz = d$, of the plane that passes through the points $(2, 0, 0)$ and $(0, 3, 0)$ and $(1, 2, 3)$ is:

____ .

1b. In the space below, clearly verify that the above 3 given points do indeed satisfy the equation that you wrote in **1a**.

2. Parameterization & Surface Area

Let m, r_1 and r_2 be fixed arbitrary positive constants with $r_1 < r_2$.

Consider the frustum $\mathcal{S}$ that is the surface of the cone:

$$z \;=\; m\,\sqrt{x^2+y^2}$$

that sits between the planes $z = mr_1$ and $z = mr_2$.
So $\mathcal{S}$ is just the side part and does NOT include the top nor bottom "lids".

2a. Make a rough sketch of the $\mathcal{S}$, labeling the two points on $\mathcal{S}$ of the form $(0, ?, mr_1)$ and $(0, ?, mr_2)$, indicating what ? are.

2b. A parameterization of $\mathcal{S}$ is:

$\Phi(r,\theta) =$ ______________________________ ,

over the domain

$D \;=\; \{(r,\theta)\;:\;$ ______ $\leqslant\; r\;\leqslant$ ______ and ______ $\leqslant\;\theta\;\leqslant$ ______ $\}$.

continued ⟹

2c. A (not necessarily normalized) normal vector to $\mathcal{S}$ is:

$\overrightarrow{T}_r \times \overrightarrow{T}_\theta =$ ______________________________ ,

and this vector has length : ______________________________ .

2d. The surface area of $\mathcal{S}$ can be expressed as the following REIMANN integral over D:

SA = ______________________________ .

2e. Integrating the integral in **2d** gives that the surface area of $\mathcal{S}$ is:

SA = ______________________________ .

continued ⟹

2f. Recall that the surface area of the curved side part of a right circular cone, with base of radius r and height h, is $\pi r\sqrt{r^2+h^2}$. Why is this in agreement with your solution to **2e**? Work Logically.

3. Green's Theorem Let a and b be fixed arbitrary positive constants.
Consider the inside of an ellipse:

$$D = \left\{ (x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} \leqslant 1 \right\}$$

along with its boundary:

$$\delta D = \left\{ (x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \right\} .$$

3a. A parameterization of δD, oriented in the counterclockwise direction, is:

$\vec{c}(t) =$ ______________________________ .

3b. Using Green's Theorem, the area of D can be expressed as the following LINE integral:

A = ______________________________ .

3c. The line integral in **3b** can be expressed as the following REIMANN integral:

A = ______________________________ .

3d. Integrating the integral in **3c** gives that the area of D is:

A = ______________________________ .

4. Stokes' Theorem

Let $\mathcal{C}$ be the ellipse in which the plane

$$z = y + 3$$

intersects the cylinder

$$x^2 + y^2 = 1 \ ,$$

oriented in the counterclockwise direction. Let

$$\overrightarrow{F}(x, y, z) = \langle 3z, 5x, -2y \rangle \ .$$

Then:

$\int_{\mathcal{C}} \overrightarrow{F} \bullet d\vec{s} =$ ____ .

A number should go on the above line; clearly indicate your reasoning below.

5. Gauss' Theorem

Let

$$\overrightarrow{F}(x, y, z) = \langle x, y, 3 \rangle$$

and $\mathcal{S}$ be the surface of the unit sphere $x^2 + y^2 + z^2 = 1$, with the outward orientation.

5a. The flux of $\overrightarrow{F}$ across $\mathcal{S}$ can be expressed as the following SURFACE integral over $\mathcal{S}$:

flux = ______________________________ .

5b. The flux is:

flux = ______________________________ .

A number should go on the above line; clearly indicate your reasoning below.

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6. Conservative Fields

A puffo is running along a path $\mathcal{C}$ given by

$$\vec{c}(t) = \langle (2-t)e^t \ , \ t \ , \ 2t \rangle \ ,$$

for $0 \leqslant t \leqslant 2$, while being subjected to a force field

$$\overrightarrow{G}(x,y,z) = \langle (1+x)e^{x+y} \ , \ xe^{x+y}+, \ 5y \rangle.$$

6a. The work done by $\overrightarrow{G}$ on the puffo as he moves along the path $\vec{c}$ can be expressed by the following LINE integral:

W = ____ .

6b. The work done by $\overrightarrow{G}$ on the puffo as he moves along the path $\vec{c}$ is:

W = ____ .

A number should go on the above line; clearly indicate your reasoning below. Some cleverness is needed. Is $\overrightarrow{G}$ conservative ? If not, can you find a conservative field $\overrightarrow{F}$ that is similar to $\overrightarrow{G}$ and then take advantage of the similarity?
