

MARK BOX		
PROBLEM	POINTS	
1	20	
2	20	
3	20	
4	20	
5 a-c	20	
Total	100	

Math 550A Prof. Girardi

Spring 97 Exam 2

3/22/97

NAME: _____

INSTRUCTIONS:

1. To receive credit you must:
 - a. WORK IN A LOGICAL FASHION, SHOW ALL YOUR WORK, INDICATE YOUR REASONING
 - b. when applicable put your answer on/in the line/box provided
 - c. if no such line/box is provided, then box your answer.
2. The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
3. As indicated on the syllabus:
 - a. allowed is a calculator
 - b. allowed are the class handouts: table of integrals, calculus formula sheet, and informal summary (along with your personal scribbles on them)
 - c. not allowed are other notes and books.
4. This exam covers (from *Vector Calculus* by Marsden & Tromba, 4th ed.) :
§ 4.3, § 4.4, Ch. 5, § 6.1, § 6.2 .

The sources of the questions on this exam are:

- (1) similar to § 4.4 # 25
- (2a) from our text: § 4.4 # 5
- (2b) from our text: § 4.4 pg. 279
- (3) similar to an example for class and homework from Chapter 5 review
- (4) similar to several homework problems from chapter 6.
- (5) typical of chapter 6.

1. Consider the vector field

$$\vec{F}(x, y, z) = \langle y \cos x, x \sin y, z \rangle .$$

1a. $\operatorname{div} \vec{F} =$ _____ .

1b. $\overrightarrow{\operatorname{curl}} \vec{F} =$ _____ .

-
- 1c. Is $\vec{F} \in C^1$? _____ (yes/no)
Explain your answer in (a few) complete sentences.

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- 1d. Can \vec{F} be a gradient vector field? _____ (yes/no)
Explain your answer in (a few) complete sentences.

2a. The figure on page 1 shows some flow lines and moving regions for a fluid moving in the plane field velocity field \vec{V} .

Where is $\text{div } \vec{V} > 0$? _____ .

Where is $\text{div } \vec{V} < 0$? _____ .

Intuitively explain your answers in (a few) complete sentences.

2b. The figure on page 1 shows the movement of a small rigid paddle wheel that is placed in moving fluid. The fluid has velocity field \vec{V} .

What can you say about the $\overrightarrow{\text{curl}} \vec{V}$? _____ .

Intuitively explain your answer in (a few) complete sentences.

3. Let R be the region in \mathbb{R}^3 that is bounded by

(1) the parabolic cylinder $x = y^2$

(2) the plane $z = 0$

(3) the plane $x + z = 1$.

Make a (rough) sketch of R and, by interchanging the order of integration, express the volume V of R as 6 different triple integrals. You do NOT need to actually perform the integration.

3-1. $V = \int \int \int 1 \, dz \, dy \, dx$.

3-2. $V = \int \int \int 1 \, dz \, dx \, dy$.

3-3. $V = \int \int \int 1 \, dy \, dz \, dx$.

3-4. $V = \int \int \int 1 \, dy \, dx \, dz$.

3-5. $V = \int \int \int 1 \, dx \, dz \, dy$.

3-6. $V = \int \int \int 1 \, dx \, dy \, dz$.

4. Let D be the region in the xy -plane enclosed by the parallelogram with vertices: $(1, 1)$, $(3, 0)$, $(4, 2)$, $(2, 3)$.

- 4a. A linear transformation T that takes the unit square

$$D^* = \{(u, v) : 0 \leq u \leq 1, 0 \leq v \leq 1\}$$

in the uv -plane onto D is:

$$T(u, v) = (\text{_____} , \text{_____}).$$

- 4b. Express $\iint_D (x^2 + 3y) dydx$ as an integral over D^* (do not integrate).

$$\iint_D (x^2 + 3y) dydx = \int \int \text{_____} dudv$$

5. Let $0 < a < b$. Let R be the region in \mathbb{R}^3 that is bounded by:

(1) $x^2 + y^2 + z^2 = b^2$

(2) $z = a$.

Let T be the transformation:

$$T(r, \theta, z) = (r \cos \theta, r \sin \theta, z).$$

5a. Then $T(D^*) = R$ and T is 1-to-1 on the interior of D^* where:

$$D^* = \{(r, \theta, z) : \text{_____} \leq z \leq \text{_____}, \text{_____} \leq r \leq \text{_____}, \text{_____} \leq \theta \leq \text{_____}\}.$$

Is T 1-to-1 on D^* ? _____ (yes/no)

Explain your answer in (a few) complete sentences with the aid of rough sketches.

continued \rightarrow

5. problem 5 ... continued ...

5b. Express the volume V of R as a triple integral:

$$V = \int \int \int dz \, dr \, d\theta .$$

5c. Calculate the above integral:

$$V = \underline{\hspace{15cm}} .$$

(Hint: think what happens as $a \rightarrow 0$ and $a \rightarrow b$).