| MARK BOX | | |
|---|---|---|
| PROBLEM | POINTS | |
| 1 | 20 | |
| 2 | 20 | |
| 3 | 20 | |
| 4 | 20 | |
| 5 | 20 | |
| Total | 100 | |

Math 550A Prof. Girardi

Spring 97 Exam 1 2/15/97

NAME: _____

SSN: _____

**INSTRUCTIONS**:

(1) To receive credit you must WORK IN A LOGICAL FASHION, SHOW ALL YOUR WORK, INDICATE YOUR REASONING, and when applicable put your answer on the LINE/BOX provided. If no such line/box is provided, then box your answer.
(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) Allowed are a calculator and the class handouts, as indicated on the syllabus. Not allowed are other notes and books.
(4) This exam covers (from *Vector Calculus* by Marsden & Tromba, $4^{th}$ed.) : Ch. 1, Ch. 2, § 4.1–4.2.

The sources of the questions on this exam are:

(1a) a typical first exam first question,
(1b) something new,
(2) from our text: § 1.1 # 16,
(3) from our text: § 2.3, example # 5,
(4) from our text: § 2.6 # 20,
(5) inspired by a most memorable CAR TALK call.

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Figure for Problem **1b.**

**1a.** Let $\vec{A} = \langle 2, -5, 3 \rangle$ and $\vec{B} = \langle -5, 1, 2 \rangle$. Let $\theta$ be the angle between $\vec{A}$ and $\vec{B}$. Let $\vec{A} = \vec{A}_{\parallel} + \vec{A}_{\perp}$ where $\vec{A}_{\parallel}$ is parallel to $\vec{B}$ and $\vec{A}_{\perp}$ is perpendicular to $\vec{B}$. Find:

| | |
|---|---|
| $\lvert\vec{A}\rvert =$ | $\cos\theta =$ |
| $\lvert\vec{B}\rvert =$ | is $0 \leq \theta \leq \frac{\Pi}{2}$ or $\frac{\Pi}{2} < \theta \leq \Pi$?____ |
| $\vec{A} \bullet \vec{B} =$ | $\vec{A}_{\parallel} =$ |
| $\vec{A} \times \vec{B} =$ | $\vec{A}_{\perp} =$ |

**1b.** In reference to the figure on page 1, fill in the blanks with either $\vec{u}, \vec{v}, \vec{w}, \vec{p}, \vec{q}$, or $\vec{r}$.

$\vec{u} + \vec{v}$ = _____

$3\vec{u}$ = _____

$-\vec{v}$ = _____ .

**2.** A parameterization of the line that passes through the points $(-5, 0, 4)$ and $(6, -3, 2)$ is:

$\vec{r}(t) \;=\; \langle$ _____ , _____ , _____ $\rangle$ ,

where:

_____ $t$ _____ .

**3.** Consider the function

$$f(x, y) = x^2 + y^4 + e^{xy}.$$

**3a.** $\frac{\partial f}{\partial x} =$ _____ $\left.\frac{\partial f}{\partial x}\right|_{(1,0)} =$ _____ .

**3b.** $\frac{\partial f}{\partial y} =$ _____ $\left.\frac{\partial f}{\partial y}\right|_{(1,0)} =$ _____ .

**3c.** An equation of the plane tangent to the graph of $z = x^2 + y^4 + e^{xy}$ at the point $(1, 0, 2)$ is: _____ .

**4.** A mountain has the shape of an elliptic paraboloid

$$f(x, y) = c - ax^2 - by^2$$

where $a$, $b$, and $c$ are positive constants. Here, the $xy$-plane lies at sea level and $f(x, y)$ represents the altitude of the mountain (measured in meters above sea level) at coordinates $(x, y)$.

**4a.** $\vec{\nabla} f =$ _____ .

**4b.** $\vec{\nabla} f\big|_{(1,2)} =$ _____ .

**4c.** At the point $(1, 2)$, the altitude is increasing most rapidly in the direction of the UNIT vector _____ .

**4d.** If a puffo releases a marble at the point $(1, 2)$, the marble would begin to roll in the direction of the UNIT vector _____ .

**5.** Houston, we have a problem. The space shuttle Atlantis is traveling along with position vector

$$\vec{r}(t) \;=\; \langle t^2, 3t^2, 4t \rangle \;.$$

If the power thrusters are turned off at time $t$, the Atlantis will coast off, with constant speed along a straight path tangent to the vector $\vec{r}(t)$. The Atlantis is almost out of fuel when astronaut John Grunsfeld notices the Mir space station off ahead of them at the position $(220, 660, 64)$. John realizes that their only hope is to turn the thrusters off, just at the proper time, so that the Atlantis will safely coast to dock with the Mir; but, John is not sure if his plan will work. So John quickly calls Tom and Ray for advice. Tom claims that John's plan will work; Ray claims that John's plan will not work. Who is right: Tom or Ray? Why?

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FIRST, WORK YOUR SOLUTION OUT ON SCRATCH PAPER. THEN BELOW MATHEMATICALLY SUPPORT YOUR ANSWER, USING COMPLETE SENTENCES TO EXPLAIN WHAT YOUR EQUATIONS ARE SAYING, AS IF IT IS A REPORT TO NASA. USE THE NEXT (BLANK) PAGE IF NEEDED. EQUATIONS WRITTEN WITHOUT EXPLANATION WILL RECEIVE NO CREDIT

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More space for your solution to question 5.