MATH 550
SPRING 1995
FINAL EXAM

Prof. Girardi

MARK BOX		
Problem	Points	
1	15	
2	15	
3	14	
4	14	
5	14	
6	14	
7	14	
Total	100	

NAME: ______________________________

SSN: ______________________________

Instructions:

(1) To receive credit you must work in a logical fashion, SHOW ALL YOUR WORK, INDICATE YOUR REASONING, and when applicable put your answer on the line (or in the box) provided.

(2) The "Mark Box" indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) Allowed are a calculator and the class handouts, as indicated on the syllabus. Not allowed are other notes and books.

(4) This exam covers (from *Intro. to Vector Analysis* by Davis & Snider, 6^{th} ed.) sections:
1.1 – 1.12, 1.14, 2.1 – 2.4, 3.1 – 3.7, 4.1 – 4.4, 4.8 – 4.12, 4.15, 4.16.

1. Let $\vec{A} = \langle 1, 2, 5 \rangle$ and $\vec{B} = \langle -4, 1, 0 \rangle$. Let θ be the angle between $\vec{A}$ and $\vec{B}$. Let $\vec{A} = \vec{A}_{||} + \vec{A}_{\perp}$ where $\vec{A}_{||}$ is parallel to $\vec{B}$ and $\vec{A}_{\perp}$ is perpendicular to $\vec{B}$. Find:

$|\vec{A}| =$

$|\vec{B}| =$

$\vec{A} \bullet \vec{B} =$

$\vec{A} \times \vec{B} =$

$\cos\theta =$

is $0 \leq \theta \leq \frac{\Pi}{2}$ or $\frac{\Pi}{2} < \theta \leq \Pi$? __________

$\vec{A}_{||} =$

$\vec{A}_{\perp} =$

2. Write a parameterization of the line through $(7, 8, 9)$ that is perpendicular to the plane given by $2x - y + 5z = 4$.

Answer: $\vec{R}(t) = \langle$ __________________ , __________________ , __________________ $\rangle$

for __________________ t __________________

3. Divergence Theorem: Consider the vector field

$$\vec{F}(x,y,z) = \left\langle\, x^3 \,,\, y^3 \,,\, z^3 \,\right\rangle \ .$$

Let D be the domain bounded by the xy-plane and the hemisphere given by $z = +\sqrt{4-(x^2+y^2)}$ and let S be the complete (bottom and top included) boundary of D. Find $\iint_S \vec{F} \bullet \overrightarrow{dS}$.

Answer: $\iint_S \vec{F} \bullet \overrightarrow{dS} =$ ____________________

Hint: switch to spherical coordinates (see handout) where $dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$.

4. Green's Theorem: Consider the vector field

$$\vec{F}(x,y) = \left\langle\, y + e^{(x^2)}\,,\ x^2 + \arctan\sqrt{y}\,\right\rangle\ .$$

Find the line integral $\int_\Gamma \vec{F}\cdot d\vec{R}$ where Γ is the square with vertices $(1,2), (5,2), (5,4), (1,4)$, traversed counterclockwise.

Answer: $\int_\Gamma \vec{F}\cdot d\vec{R}$ = ______________________________

5. Stoke's Theorem: Consider the vector field

$$\vec{F}(x, y, z) = \left\langle -3y^2 \, , \, 4z \, , \, 6x \right\rangle \ .$$

Find the line integral $\int_C \vec{F} \cdot d\vec{R}$ where C is the triangle with vertices $(2,0,0), (0,2,1), (0,0,0)$ oriented counterclockwise when viewed from above.

Answer: $\int_C \vec{F} \cdot d\vec{R} =$ ______________________________

Hint: *special case*

6. Consider the vector field

$$\vec{F}(x,y,z) = \left\langle\, ze^{xz} \,,\; x \,,\; xe^{xz} + y^2 \,\right\rangle \;.$$

a) The domain D of definition of $\vec{F}$ is ____________________________ .

b) Is $\vec{F}$ conservative in D ? Why or why not?

c) Find the line integral $\int_C \vec{F}\cdot d\vec{R}$ where C is the helix from $(1,0,0)$ to $(1,0,2\pi)$ given by $\vec{R}(t) = \langle \cos t, \sin t, t\rangle$. Hint: there is an easy (clever) way..... as in homework

answer: $\int_C \vec{F}\cdot d\vec{R} =$ ____ .

7. A puffo is running along the curve $y = e^x$ in such a way that its abscissa (ie. the x coordinate) is increasing in time. The puffo's speed (i.e. *scalar* velocity) is a constant 4 (ft/min) and his *scalar* acceleration is a constant 10 (ft/min ^{2}). Express his velocity vector and acceleration vector as a function of the abscissa (ie. as a function of x and *not* time t).

ANSWER: $\vec{v}(x) = <$ __________________________ , __________________________ $>$

ANSWER: $\vec{a}(x) = <$ __________________________ , __________________________ $>$

Hints: Note that the derivative (with repect to time t) of the position vector (as a function of time) gives a vector tangent to the curve. The derivative (with repect to x) of the position vector (as a function of abscissa) also gives a vector tangent to the curve. A vector is determined by a direction and a length. Next note that if, as a function of time t , his velocity vector looks like $< v_1(t), v_2(t) >$, then $[v_1(t)]^2 + [v_2(t)]^2 = 16$. Differeniate both sides of this equation with respect to t to see what it says about $v(t)$ and $a(t)$. PRESENT YOUR SOLUTION IN A LOGICAL ORDERLY WAY! MARKS WHICH APPEAR TO ME TO BE RANDOM WILL RECEIVE NO POINTS! The next page is blank to provide you with more space if so needed.